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http://www.tenman.eu/Wirtschaft-Politik/Wirtschaft-RIKES/Keynes-und-Komplexitaet/

Keynes in the light of modern complexity theory

by graduate economist Leander L. Hollweg

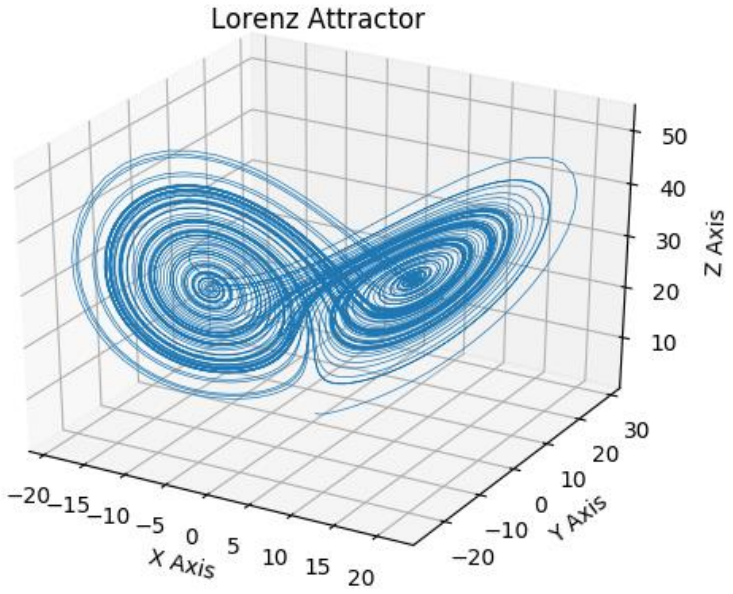
Could it be that John Maynard Keynes left us a hidden message in his "General Theory"? Hidden because he had come across insights that he himself only guessed at but did not explain in detail. Insights into the dynamics of the economic process that even his genius could hardly put into words? And that this partial understanding was due to the fact that the science of his time did not yet have certain mathematical methods and technical means at its disposal, which were only developed in the 1970s? So it could be that Keynes was not only an outstanding economist in terms of the problems of his era, but that his work already pointed far beyond the intellectual horizon of his time. And that this has been misjudged even by his most advanced epigones. In the light of modern complexity science, a reading of his work provides evidence for this thesis.

Keynes' analysis, which succeeded in making neoclassical equilibrium analysis appear as a special case of his own new model, would thus also have to be seen as only a subset of a more comprehensive explanation. Keynes' theory is mainly concerned with the area of economic life in which built-in stabilisers regularly lead back to justifiable, albeit not optimal, system states. With the exception of what he called the "liquidity trap", however, Keynes refrained from examining system behaviour at the limits of the stability zone and beyond. More recent findings, however, allow this and recognise in Keynes' theses the basic pattern of an even more comprehensive fundamental property of complex systems.

The practical benefits of this insight could be twofold. On the one hand, there should be a growing awareness that the level of prosperity of an economy is exposed to dramatically higher risks than we have assumed so far. On the other hand, it is becoming apparent that economic policy could be more successful with intelligent, minimally invasive measures than with large economic stimulus programmes.

The theory of complex systems (often referred to as dynamic or non-linear systems) has been known as "chaos theory" since the mid- to late nineteen-seventies and has since been thoroughly misunderstood. In economics, this theory has so far only been used occasionally. For some years, banks and stock market speculators tried to use the theory to predict stock and foreign exchange prices. After it became apparent that this could only be done inadequately, these attempts were

largely abandoned at the end of the nineteen-nineties. In the meantime, however, the laws recognised for dynamic systems have found practical application in many ways, above all in physics, electrical engineering, heart medicine, weather research, biology and sociology. They have enriched mathematics and contributed to a new branch of this discipline. The most well-known symbol of the field is the Lorenz attractor, which is also associated with the now almost proverbial "butterfly effect" because of its shape.



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A "definition" of the properties of complex systems in the form of a short, concise statement has hardly been achieved so far. However, it is important to understand that the "chaos" found in them is not to be understood as "completely wild disorder". The "scientists who studied chaotic dynamics made the discovery that a creative process could be attributed to the irregular behaviour of simple systems. It gave rise to com-plexity; highly organised patterns that were sometimes stable and sometimes unstable, sometimes finite and sometimes infinite, but which always possessed the fascination that living objects have."

Part of this fascination is the observation that "life" has the property of "emergence": it cancels out its own structure through reflexivity and adaptation. Physics Nobel Prize winner Murray Gell-Mann introduced the term "complex adaptive systems" for this.

A dynamic system is therefore an entity that exhibits chaos and stability at the same time and can be very robust in this duality. A wonderful example of this has proved to be the famous "big red spot" that can be seen on the planet Jupiter or in its atmosphere. It is a tremendous meteorological phenomenon, a kind of storm in the midst of an even more tremendous storm, which, despite the inferno within and around it, persistently finds itself: the peculiar structure of this "island in chaos" remains untouched. At the same time, "the stain ... is a system that organises itself, generated and regulated by the same non-linear twists that cause the unpredictable unrest around it. It is a >>stable chaos<<" .

Chaos and instability are thus by no means the same thing. Thus it was shown: "A chaotic sys-tem could be stable if its special characteristic of instability persisted in spite of minor disturbances. ...One could disturb this system, shake it, stir it, interfere with its movements, but afterwards, when everything had calmed down again and the short-term disturbances had died away like an echo in the mountains, the system would return to exactly the same pattern of irregularity that it had exhibited before. It was locally unpredictable and globally stable.

Nobel Prizes and other prestigious scientific awards have been given to scientists researching in this field. Certain relationships have now been recognised by these researchers as "universal" truth. Only economics seems to be largely untouched by this universal truth! Keynes, however, in the view put forward here, anticipated something of this truth. He led us on a trail, the scent of which, however, his guild did not pick up. We economists were blind and deaf to his finger-pointing and warnings. An event like the recent international financial crisis might otherwise have been prevented, and it might have been much easier for us to deal with the crisis.

For if this view is correct, the very recent results of complexity research indicate that it may not take a huge stimulus package to rebound an economy even if it has fallen into a downward spiral. Small but very well-timed and targeted economic stimuli would be sufficient to recalibrate and stabilise economies.

Equally, however, there is obviously a danger at a certain point that a tiny cause could extremely destabilise the entire economic system. There is a saying that no one should care if the famous sack of rice falls over in China. Today, for example, the refusal of a single taxi driver in Shanghai to accept a dollar bill in payment instead of yuan could trigger a worldwide financial crash. Small cause - big effect! The recent revolution in Tunisia, completely unexpected by all political observers, was triggered by reactions to the fate of a vegetable vendor who had been harassed and slapped by the police. Such upheavals become possible when "the system" leaves a containable zone of stable behaviour. Keynes knew that such situations could be possible, and he was cautious about it. Too cautiously, apparently, so that it was hardly noticed and, above all, not taken seriously enough.

Keynes emphasised: "In particular, it is an outstanding characteristic of the economic system in which we live that, though prone to serious fluctuations in output and employment, it is not dangerously unstable. Fluctuations may set in violently, but they seem to run their course before they have risen to great extremes, and our normal destiny is a condition neither hopeless nor satisfactory."

But he also notes:

"The position of equilibrium is thus influenced by these* three repercussions; and in addition there are other repercussions. Moreover, there is not one of the factors mentioned that does not have a surprising tendency to change, and sometimes even to change fundamentally. Hence results the extraordinary complexity of actual development" .

to finally concede

"It is not impossible that there could be an area where perhaps real instability prevails."

and to warn

"But we must not conclude that the middle position, which is thus determined by >>natural<< tendencies, is for that very reason brought about by necessary laws. ... The unlimited validity of the above conditions is a fact of observation from the world as it is or has been, but not a necessary, unalterable principle."

Keynes thus notes the "extraordinary complexity of actual development". However, a complex system - as research has since proven - is something different from a complicated system. Up to now, we economists have certainly regarded economic relationships as a complicated web of interacting influencing factors. The task of economics was to isolate the essential determinants from this tangle, to regard them as strong influencing factors, to relate them to each other and to construct a relationship model from this set of independent variables. The number of acting factors in this model is reduced and their interrelationships thus become manageable, their effects and interactions can be explained, and at the end there is usually the social product or the level of employment as the dependent variable, which can be clearly quantified from the model at any time t0 to tn. Economic modelling is therefore reductionist and leads to simplifications that nevertheless - or precisely because of this - generate regularity, exhibit stable patterns and allow reliable predictions with acceptable fuzziness. The objection that reality is actually more diverse, that the models are therefore alien to life and therefore wrong, cannot harm this science. It is precisely important to reduce the degree of complexity and to bring out of the cacophony of millions of individual decisions by economic subjects a clarity that would otherwise be impossible to achieve, because otherwise we would have to despair in the face of the diversity of life.

Keynes also proceeds in this way. At least at first. Up to the middle of his main work, he peels out the significant influencing factors of the economic process chapter by chapter in razor-sharp analysis and with precise definitions. In the 18th chapter, not at the end but in the middle of his book, he puts the puzzle together. In Section I, he first explains the structure of his model in order to "clarify which elements in the economic system we initially take as given, which are the independent and which the dependent variables of our system". The result is a structure of 5 basic elements, namely the three basic psychological inclinations of economic citizens (propensity to consume, liquidity preference, expected return), the wage rate and the money supply. At the end of this first section, Keynes states that he is convinced that he has thus succeeded in identifying "those factors which chiefly determine our object of investigation". It is then only a small step from economic analysis to economic policy: "Our final task would be to find out those variables which can be consciously controlled and regulated from the central point of the economic system in which we at present live."

In the second section of Chapter 18, however, Keynes now takes an unusual step. He summarises the argumentation of "the preceding chapters" and in this section he treats " the influencing variables in the reverse order in which we introduced them." In Section I he had already referred several times to interactions between different variables. In the following, this back-reference leads linguistically to complicated loops that make his work and especially this decisive 18th chapter difficult to read, not only in the inadequate German translation from 1936, but already in the English original. Those who manage to immerse themselves emphatically in the language recognise a certain rhythm, a gentle swing, a flow of strands twisting into one another. Now "the amount of new investment" is at the centre of the action, and "this means that

- the material supply conditions in the capital goods industries,

- the state of confidence as to the likely rate of return...

- the psychological attitude towards liquidity

- and the money supply (in wage units)

interact with each other to determine the level of new investment."

The first two variables together, as the expected surplus of the investment result in relation to the investment costs, correspond to the "marginal efficiency of capital" (GLFK).

The development of this parameter constantly "struggles" with the interest rate for which debt capital can typically be procured on the capital market for investment purposes or which could be achieved at average conditions through alternative financial investment. In this view, an investment impulse leads to increased demand for money (liquidity prevalence for transactions) via the known multiplier and accelerator effects through price increases and thus to interest rate increases that initiate a new downturn. The economy would only enter a prolonged dynamic equilibrium path with continuous income growth if the interest rate were to fall, and fall faster than the GLFK naturally falls.

"The" interest rate does not exist in Keynes' analysis. It is not only that a large variety of interest-bearing securities of the most diverse risk classes and maturities are naturally traded on the markets: here, an average is simplistically thought of, moreover independent of the maturity. But Keynes distinguishes the interest rate associated with real investment (one could almost say the rate of profit) from the interest rate that can be obtained from capital investments on the financial markets. In reality, every bank customer knows the real but mirror-image equivalent of this phenomenon: the credit interest rate or investment interest rate for savings and the debit interest rate for loans taken out. Both interest rates are coupled with each other, but also move against each other, and can hardly be sharply distinguished from each other in the practice of the financial markets as well as theoretically: they are interdependent.

However, while the GLFK and the lending rate i depend on real economic developments, the movement of interest rates on financial investments is, according to Keynes, a function of money supply and liquidity preference, whereby, in a tautological definition, liquidity preference is in turn a reciprocal function of this interest rate. The irritating factor in this relationship is the tendency to hold money for speculative purposes (L2): "Experience indicates that the aggregate demand for money to satisfy the speculative motive usually shows a continuous response to gradual changes in the rate of interest. In general, according to Keynes, "L2 depends mainly on the relation between the current rate of interest and the state of expectation", from which it follows that the (financial) interest rate r is "a highly psychological phenomenon" or ultimately a result of "convention" in the sense of a generally widespread expectation.

Insofar as expectations about future changes in interest rates influence cash holdings, Keynes points out that the mathematical calculations of financial investors are governed by whether the interest rate differential is greater than, less than or equal to the square of the current interest rate: "For example, if the rate of interest on long-term debt is 4 per cent, it is preferable to sacrify liquidity unless ... it is feared that the long-term rate of interest may rise faster than by 4 per cent of itself per annum, i. e. greater than 0.16 per cent of itself per annum.

If one looks at Keynes' work with the understanding that the Berlin FU professor Hajo Rie-se at the Institute for the Theory of Economic Policy encouraged his students to have, namely as a "liquidity preference theory", in which the interest rate is not the price of capital but the price of giving up liquidity, then the alarmed interest of economists should actually have started at this passage long ago. With this maxim of retention, Keynes provides an explanation for the rule according to which bond investors increase their cash holdings or reduce their liquidity in exchange for fixed-interest securities. The mathematics behind this will lead us back in condensed form to the following

description of the system, as described in the appendix under the heading "Hicks and Keynes: Squaring the Interest Rate". In the meantime, I would like to assert:

The non-formalised quadratic equation is the pivot of the whole of Keynes` theory.

But first, let us return to the context of the system variables that Keynes presents in the 18th chapter. Keynes thus dissects the moving heart of the economy as the dynamics of interdependent interest rates, some of which sound in unison with each other, some of which work against each other and "eat" each other. Built-in stabilisers of the system (linkages of the main variables 1.) Propensity to Consume, 2.) GLFK and 3.) capital-market interest rates) with generally low elasticity effects ensure (to put the quotation already given once again in its exact context) "that it is an outstanding characteristic of the economic system in which we live that, though prone to serious fluctuations in output and employment, it is not dangerously unstable." But Keynes qualifies this statement again shortly afterwards: "Since all these facts of experience do not follow with logical rigour, it must be supposed that the environment and the psychological peculiarities of the modern world are such as to produce such results."

One can hardly assume more than self-appeasement in this. For Keynes had already noted in Chapter 14 that there had indeed been a "complete breakdown of stability" in his lifetime, namely in Russia and in Central Europe in the form of hyperinflation and, conversely, in 1932 in the USA in the form of an illiquidity crisis in which hardly anyone wanted to part with money holdings, whatever the conditions.

Everywhere, therefore, we encounter the spectre of dramatic instability with Keynes, which can only be shooed away with difficulty, but is also never seriously addressed.

If scientific knowledge had spread as rapidly through the scientific community in the 1920s and 30s as it does today, Keynes would have recognised a startling analogy: His system of competing and interacting interest rate caluclations corresponds to the Lotka-Volterra equations approach.

However, Keynes was probably not aware of these mathematical works because their "discovery" had been made only ten years before the completion of his economic theory in a completely different field of science. However, as we can see, it cannot be ruled out that he may have had or could have had knowledge of them. (On this I make remarks in Appendix 2, "Keynes as a Capitalist: Bonds or Shares?")

In this case, he would simply have spared his readers mathematics that was difficult to understand and deliberately replaced it with linguistic statements, (at least as far as this seemed possible to him, because Keynes was also an admirer of the philosopher Ludwig Wittgenstein, whom he had invited to the honeymoon domicile in Sussex during his honeymoon, of all times, to the dismay of his young wife Lydia. See on this: https://www.telegraph.co.uk/money/f-and-c-investment-trusts/john-maynard-keynes/ there: key point 9, footnote 10.)

Whether or how intensive Keynes' knowledge of Lotka's equation was is also decisive for statements about Keynes' relationship to Hicks, or whether Keynes plagiarised Hicks' original findings without reference to the source, see Appendix 1).

Permitted by a Common Creative Attribution licence, it is explained here in detail with the help of Wikipe-dia (=font Times New Roman) what these equations are about:

"The Lotka-Volterra equations, also known as predator-prey equations, are a system of two non-linear, coupled first-order differential equations and describe the interaction of predator and prey

populations. Predator and prey refer to two classes of organisms, one feeding on the other.[V 1] The equations were formulated in 1926 by Vito Volterra[1] (an Italian, LH) and, independently, in 1925 by Alfred James Lotka[2] ."

The equations are

$$\frac{dN_1}{dt} = N_1(\epsilon_1 - \gamma_1 N_2), \qquad \frac{dN_2}{dt} = -N_2(\epsilon_2 - \gamma_2 N_1)$$

with the terms[V 2]

N1 = N1(t) number of prey organisms time-dependent

ε1 > 0 Reproduction rate of the prey constant without disturbance and with large proximity supply

N2 = N2(t) Number of predators time-dependent

ε2 > 0 Mortality rate of predators when no prey is available constant

γ1 > 0 Feeding rate of predators per prey species = mortality rate of prey per predator constant

γ2 > 0 Reproduction rate of predators per prey organism constant.

The Lotka-Volterra equations are an important foundation of theoretical biology, and therein especially of population dynamics. The predators and prey do not necessarily have to be animals or individual species; in principle, the model can be applied to guilds.

A guild is a group of species that use similar resources in a similar way, regardless of their degree of relationship. ... because of this common use of the same resources, there is inevitably competition in guilds between representatives of different species (interspecific competition). Accordingly, niche differentiation is to be expected between these species if both are to coexist.

The applicability of the Lotka-Volterra equations depends on the extent to which the justification of the mathematical model applies in the individual case.

Justification of the mathematical model

Volterra justifies his system of equations as follows [V 3]:

- Let the population numbers of the prey and the predators be denoted by N1 and N2, respectively.

- Let the undisturbed growth rates per time unit dt be λ1 and λ2, with the antecedents not yet fixed.

- The (mean) number of encounters between prey and predator per unit time dt is αN1N2 with a positive real number α, which is assumed to be constant within a biotope but generally depends on the biotope.

- A sufficiently large number n of encounters have an average effect βi on the population number Ni. In the case of prey organisms, this is readily apparent: an encounter with a predator leads with a certain probability to the prey being eaten. In contrast, the effect of an encounter on the number of predators is only indirect, but in any case positive; for the modelling, such an effect on the population number is also assumed for the predators.

For the mathematical treatment of Lotka-Volterra systems, the somewhat simpler notation [M 1] is usually used today

$$\frac{dN}{dt} = N(a - bP), \quad \frac{dP}{dt} = P(cN - d),$$

where a,b,c,d are positive constants and N(t) denotes the number of prey and P(t) the number of predators.

Keynes' analysis corresponds to this predator-prey scheme:

In these equations, understand

- N as the lending rate i

- and P as the investment rate r

and

- a as the given factor of change of the lending rate i due to the real multiplier effects from investment activity (effect in general = tendency to increase interest rates due to money shortage at constant money supply)

- b as a given "psychological" expectation factor (absolute amount) for a change in the investment interest rate r as a function of rising lending rates i (effect as a rule = speculative cash falls and demand for financial securities/securities increases securities prices rise and yields r fall)

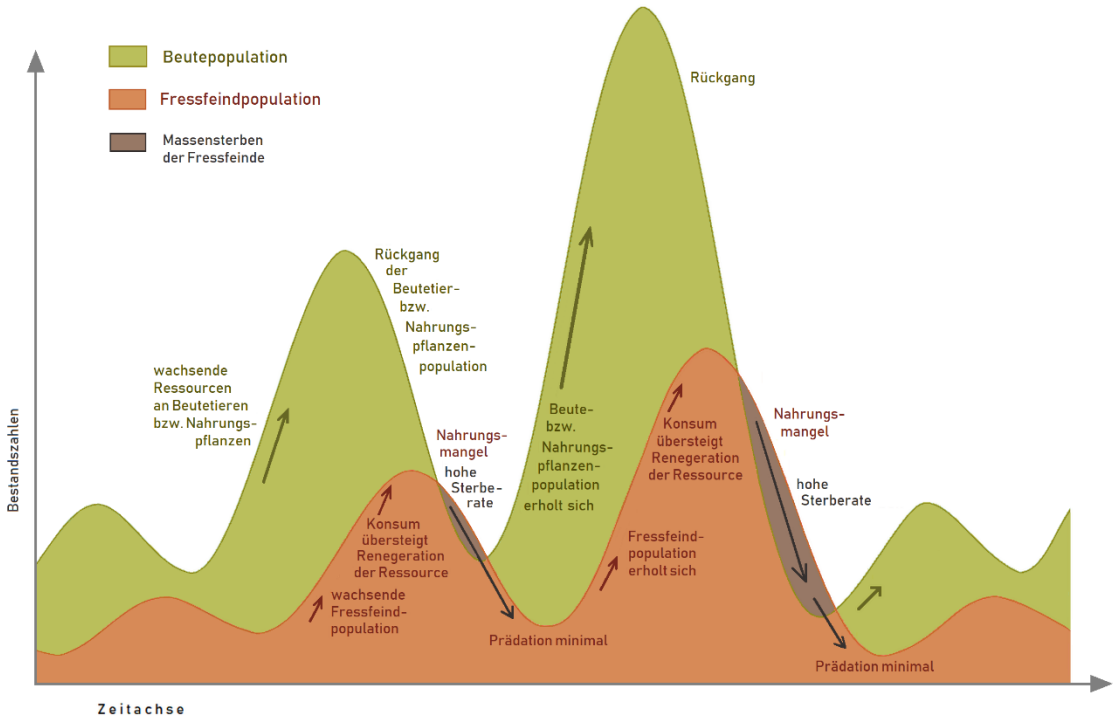
- c as a given "psychological" expectation factor for a reduction in real investment returns (GLFK) as the business cycle progresses (effect as a rule: lending rate i tends to fall as a result)

- d as the given elasticity factor of the investment interest rate r to the change in itself (effect usually reciprocal, e.g. rising investment interest rates cause reduced cash holdings thus rising securities prices thus falling values for the financial investment result r in the case of new financial investments; however, rising values for r in the case of premature liquidation of financial investments acquired in earlier periods).

This model shows quite well which logically intertwined expectations dominate the calculations of financial investors after an investment cycle has begun. In Germany, for example, we are currently (=year 2011) experiencing confused conjectures about the resulting development of the guaranteed interest rate for life insurance policies, after the Solvency II Directive requires insurance companies to invest the capital of insurance customers predominantly in increasingly risky government bonds in the future. Portfolio effects and new investment results do not allow for a serious intuitive forecast in the short and medium term.

In the Lotka-Voltera-Keynes equation, an interest-rate-increasing impulse is counterbalanced by three feedback effects that have an interest-rate-lowering effect. Falling interest rates, in turn, could increase the propensity to invest and put the system on a growth path. So this is possible, albeit the exception.

As a result, the predator-prey dynamic typically develops into the following process:



https://upload.wikimedia.org/wikipedia/commons/4/44/Nat%C3%BCrliche_Regulation.png

Plotting the populations over time gives the picture of a sine-like oscillation with a phase shift between the predator and prey populations. Note the crash of the populations to a much lower level after passing through the 3rd cycle.

In the meantime, a simplification of the Lotka-Volterra equations can be derived from the empirical studies on population development according to Pierre-François Verhulst. This development is represented by the logistic equation.

The logistic equation was originally introduced in 1837 by Pierre François Verhulst as a demo-graphic model. The equation is an example of how complex, chaotic behaviour can arise from simple non-linear equations. As early as 1825, Benjamin Gompertz presented a similar equation in a related context.

The basic problem expressed by the equation has actually been known to economists since the predictions of Robert Malthus: The population grows faster than its food resources and is thus periodically decimated. But it was not until the 1950s that some ecologists, such as W. E. Ricker and Robert May, tried to make variants of this special equation applicable to practical questions, e.g. the development of fish greens.

The historian of science James Gleick notes the problems encountered in solving the equations: "Strangely enough, the continuous sequence of numbers produces an irritating behaviour - quite a pain for someone who has to do his calculations (at that time) with a hand crank. ...Obviously, none of these early ecologists had the energy to keep producing numbers that would not do them the favour of coming to an end. If the population figure jumped back and forth incessantly, the ecologists

were convinced that it revolved around a hidden state of equilibrium. The thought that there might not be an equilibrium did not occur to the ecologists at all." …

"As soon as the models contradicted the authors' knowledge of the behaviour of real populations, some missing characteristic provided the explanation for this discrepancy: for example, the age structure within the population in question, restrictions due to the terrain or certain geographical conditions, or even the difficulty of having to calculate with two sexes at the same time. ... The stable solutions were considered the really interesting ones. Order was a kind of self-reward. After all, it was a hard business to find out the right equations. Nobody therefore felt like wasting time on a field of work that led to error results and lacked the desired stability."

It was only as a result of a landmark paper by theoretical biologist Robert May in 1976 that the logistic equation became widely used.

The mathematician James Yorke, who worked with May, later explained: "If you want to write down the solution of a differential equation, it is necessarily not chaotic, because in order to write it down, you need regular invariables - factors that remain constant, like angular momentum" . ... Solvable systems are those that are demonstrated in textbooks. They show the desired behaviour. Most differential calculus, on the other hand, are not solvable. Only very few scientists are able to account for the fact that the solvable, ordered linear systems represent the actual exceptions. ... Differential equations were an invention for a computerless world in which scholars were still dependent on making calculations of natural events with paper and pencil. They represented reality as a continuum and would glide abruptly from one point to another distant point, and from time to time, rather than being divided into discrete coordinate steps or time increments. With the computer, on the other hand, an "extended" arithmetic operation is possible: in small step sequences, the values for a dependent variable xt0 are calculated from an equation and used again as the initial value for the next arithmetic step to determine xt1. The result is not a solution of the equation in the sense of a simultaneously resulting numerical or coordinate value, but the movement of a sequence from the set of complex numbers in a "phase space" with TIME as an indispensable dimension.

"Devil's work" - the logistic equation

With this background, let's now take a closer look at the logistic equation. Again, Wikipedia should help us : Mathematical regularities are sought which represent the development of a population in a model-like way. From the size Xn of the population at a certain point in time, the size Xn + 1 after a reproductive period (e.g. after one year) is to be inferred.

The logistic model takes two influences into account:

1. the population increases geometrically through reproduction. In the following year, the number of individuals is larger than the current population by a growth factor qf. 2.

2. starvation reduces the population. The number of individuals decreases depending on the difference between their current size and a theoretical maximum size G with the proportionality constant qv. The factor by which the population decreases therefore has the form

$$q_h = (G - X_n) \cdot q_v .$$

In order to take both processes into account when calculating the population in the following year, the current population Xn is multiplied by both the multiplication factor qf and the starvation factor qh. The logistic equation is then obtained. This gives the logistic equation

$$X_{n+1} = q_f \cdot q_v \cdot X_n \cdot (G - X_n)$$.

To simplify the following mathematical investigations, the population size Xn is often given as a fraction xn of the maximum size G :

$$x_{n+1} = \frac{X_{n+1}}{G} \quad ; \quad x_{n+1} = \frac{X_{n+1}}{G}$$

G, qf and qv are combined to the number

$$r = G \cdot q_f \cdot q_v$$.

A common notation for the logistic equation is the following:

$$x_{n+1} = r \cdot x_n \cdot (1 - x_n)$$

Here K is the capacity of the biotope, i.e. the population corresponding to the fixed point of the dynamics if r is chosen appropriately.

The mathematical model]

This gives: = **$r\ (x_n - x_n^2)$**

xn is a number between 0 and 1. It represents the relative size of the population in year n. The number x0 represents the population size in year n. The number x0 therefore represents the starting population (in year 0). r is always a positive number, it represents the combined effect of reproduction and starvation.

Let us again translate this equation into the language of Keynesian economics: the term r in this equation represents a combined given expectation of the market in the sense of a factor of change. If we replace the divergence of the two antagonistic interest rates i and r in the Keynesian context by the theoretical one-unit interest rate x, which interacts completely with itself and thereby depends on the interplay of liquidity demand with the interest rate, we discover that the quadratic term in the equation corresponds to Keynes' reference to investor calculi that change with the square of the initial interest rate. In purely practical terms, interest rates lie in the interval between 0 and 1 (100%).

However, the apparently simple equation r (x-x2) contains a real mathematical devil (Benoit Mandelbrot spoke of a "devil's polymer"):

For different r, the following behaviour can be observed for large n. This behaviour does not depend on the initial value. This behaviour does not depend on the initial value, but only on r:

- With r from 0 to 1, the population dies in any case.

- With r between 1 and 2, a limit value is reached. The approach to the limit value is monotonous.

- With r between 2 and 3, the population approaches its limit value in a wave-like manner, i.e. the values are alternately above and below the limit value from a certain n onwards.

- With r between 3 and (about 3.45) the sequence alternates between the two environments of two clustering points for almost all starting values (except 0, 1 and).

- With r between and about 3.54, the sequence alternates between the environments of four cluster points at almost all starting values.

- If r is greater than 3.54, first 8, then 16, 32 etc. cluster points appear. cluster points appear. The intervals with the same number of cluster points (bifurcation intervals) become smaller and smaller; the length ratio of two successive bifurcation intervals approaches the Feigenbaum constant. This constant is also important in other mathematical contexts. (Numerical value: $\delta \approx 4.669201609102990671853203820466201617258 1...$).

- At r approximately 3.57, chaos begins: the sequence initially jumps around periodically between the environments of the now unstable cluster points. As r increases, these intervals merge so that their number is halved in the rhythm of the Feigenbaum constant until there is only one interval in which the sequence is chaotic. Periods are then no longer recognisable. Tiny changes in the initial value result in a wide variety of subsequent values - a property of chaos.

- Most coefficients between 3.57 and 4 lead to chaotic behaviour, although for certain r, there are cluster points again. For example, in the vicinity of r = 3.82, as r increases, there are first 3, then 6, 12, and so on. cluster points. Likewise, there are r-values with 5 or more cluster points - all period durations appear.

- For r greater than 4, the sequence diverges for almost all initial values and leaves the interval [0;1].

This transition from convergent behaviour via period doublings to chaotic behaviour is generally typical for non-linear systems, which show chaotic or non-chaotic behaviour depending on a parameter.

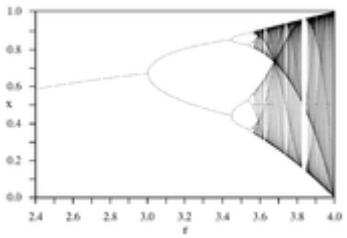
An extension of the value range to the complex numbers leads to the Mandelbrot set after a coordinate transformation.

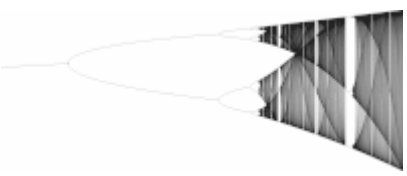
The associated dynamics can be illustrated by means of a so-called fig tree diagram (see below). An important role is played here by the Feigenbaum constant discovered by Mitchell Feigenbaum as early as 1975.

Graphical representation

The following bifurcation diagram, known as the Feigenbaum diagram, summarises these observations. The horizontal axis indicates the value of the parameter r and the vertical axis the cluster points for the sequence xn.

Bifurcation diagram of the logistic equation





High-resolution version without scale

Relationship with the Mandelbrot set (after coordinate transformation)

Keynes deserves special credit for proving that developments on the financial markets control the real economy, i.e. that they determine the results of the goods and labour markets - and not vice versa, as the "neoclassically" founded causal chains claim. This, at least, is what I understand by the view of the "Berlin School" of Keynes interpretation founded by Prof. Hajo Riese. Thus, the "liquidity trap" recognised by Keynes refers to the phenomenon when money is no longer offered for investment when interest rates fall and thus tends to remain withdrawn from the economic cycle. The conditions of the crisis of confidence that prevailed on the financial markets in 2008/2009 after the collapse of Lehman Brothers, so that the market for interbank loans dried up, were thus astonishingly well anticipated by Keynes as the "possibility" of an economic shock paralysis or derived from observations of the course of the economy in the USA in 1932.

In the light of complexity theory, however, this situation is only one example of many very different unusual system states that can result from market developments. And the tricky thing is that, on the one hand, these states cannot be predicted in principle, but on the other hand, they can nevertheless follow repetitive patterns. These patterns are expressed in the shorter and longer waves of economic development, and "similar" repetitions of apparent singularities are not only conceivable but probable. The sudden drop in share prices of around 60% that occurred on the world's stock exchanges within only 20 minutes on 6 May 2010 (and which was almost made up for two hours later) must therefore be understood as a portent.

Economic historians now believe that behind the "rise and fall of the great powers" they can recognise the work of 150-year-old "intergrals". The Chinese, too, are accustomed to viewing their history in cycles of this amplitude - and in die-serious certainty, since the Maoist revolution, consciously see themselves again on an ascending branch of their historical destiny. It is therefore necessary to familiarise oneself with the idea that economic systems can also experience a total crash that throws them completely off course and from which they cannot recover even in the long term. The (Spanish-dominated) Habsburg Empire got into such a situation in the 16th century: after the discovery of America around 1500, Europe was "blessed" with a massive supply of precious metals, which in the end was not a blessing but an inflationary curse. The supposed wealth encouraged the over-expansion of the empire, unhealthy lending in the form of the so-called "juros" (government bonds) and the nonsensical decades-long warfare against the renegade Netherlands. In the end, Europe collapsed in the horrors of the 30 Years' War; Habsburg (Spain) had to declare state bankruptcy three times between 1557 and 1650 (in 1557 at the same time as France, which subsequently caused the downfall of the Fugger trading and banking house; and in 1596 and 1607).

With this real-life example in mind, the inflationary flooding of Western economies with central bank money as a supposed Keynesian solution to the crisis should be reconsidered very critically!

It should not be difficult to realise that the dynamics inherent in the logistic equilibrium, which we succeeded in transferring from biology to economics with the help of Keynes' perceptive view of the money markets, are precisely those dynamics contained in every market relationship in which prices can form under free competition between numerous suppliers and demanders. This is the reason why Benoit Mandel-brot was able to derive his famous set of figures named after him from the long-term development of US cotton prices - which probably few economists know. The classic spider-web theorem of the first semester textbooks, in which supply and demand always move towards a central intersection in only quasi-dynamic analysis, has thus had its day!

The world-famous stock market speculator George Soros formulated this insight as early as 1994. "Under the title of a >>theory of reflexivity"<< he describes positive feedback processes that come about because not only speculation but all financing influences the facts to which it purports to react: "It goes without saying that the security underlying a loan is strengthened, i.e. made more secure, by the fact that the loan is granted. Of course, share prices are not merely passive reflections of the well-being of a company, but rather active influences on that company. ... Much more important is the realisation that the claimed re-flexivity already applies to every market price. Every market price is a valuation which, as such, influences the course of events - and influences them unpredictably..."

However, the mathematical structure of the problem has now become clearer. A number of recent analyses have looked fundamentally at the mathematical-immanent workings of the Cobweb theorem. The result: "If the slope of the price function (in terms of amount) is greater than that of the diagonal (45 degree line) then .... there is no equalisation (over time). This is therefore a divergent system." . "Actually, one should formulate more elegantly: if the supply reacts more elastically to the price than the demand". Market equilibria are thus possible, but not guaranteed.

Special economic studies have meanwhile come across antagonistic-interacting relationships in many places, which could therefore be modelled with logistic equilibria. For example, the substitution of the production factors capital and labour. The well-known trade-off between inflation and unemployment, which is sometimes true but then again cannot be observed empirically, might also fit into the category of such problems, I suspect.

The post-doctoral thesis by Andreas Liening, an economics didacticist at the University of Münster, from 1998 gives a good overview of "Recent developments in the theory of non-linear dynamic systems and the significance for economics". One gets the impression, however, that the economic mainstream, at least in the USA and Europe, is largely unaffected by these "recent developments". So far, it seems to be rather thematically limited special studies that make use of the mathematical tools of "chaos theory" or the theory of non-linear, dynamic systems and thus produce fruitful results.

Some of these studies deal with the market for pigs. This is an interesting area of study because economists believe that they have analysed this market particularly well. With the theory of the "pig cycle", which is derived from a quasi-dynamic Cobweb model, a theorically sound tool is supposedly available that can accurately explain real processes and is therefore also verified as a theory. On closer examination, however, both the model theory and the empirical analysis arrive at completely confusing observations that run counter to the pig cycle theory and completely different model

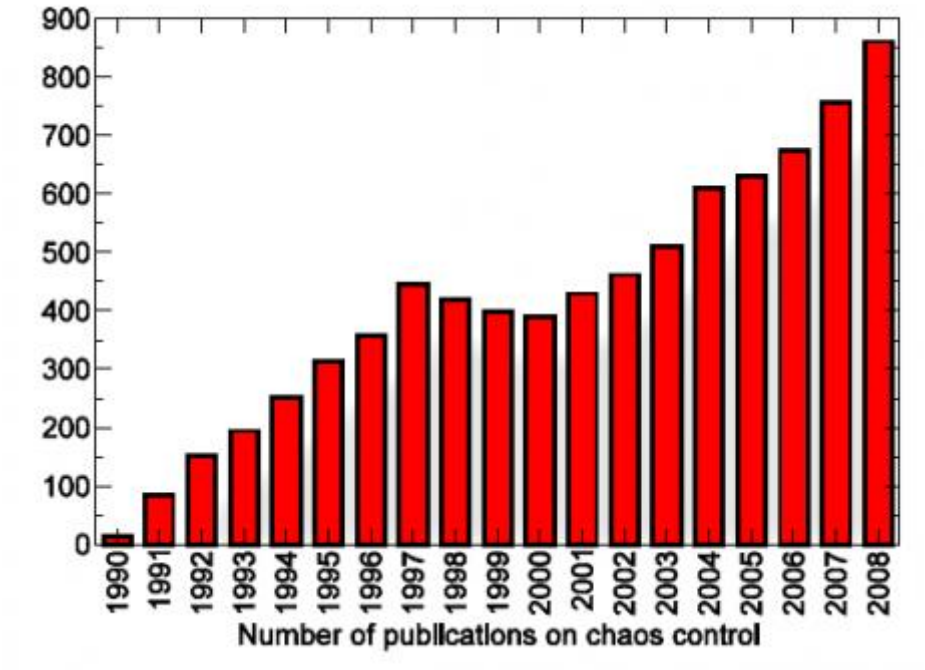
courses. It increasingly appears that we economists have little accurate knowledge of even the most basic foundations of our science!

I have the impression that Japanese and increasingly Chinese authors often appear in studies with a complexity-theoretical approach. It could be that non-linear considerations correspond more to Asian thinking, in which "everything is interconnected with everything else", than to the Western tradition of thinking, which is often characterised as mechanistic and traced back to the Newtonian world view. However, I would like to point out that this interpretation of Newton is based on a fundamental misunderstanding that is unfortunately firmly anchored in Western general knowledge and is still taught in our schools today. The misconception is that Newton proved that the Earth orbits the Sun and that this orbiting occurs in a precisely calculable, uniform path. The truth, however, is that Newton saw an interactive system in the attraction and repulsion of two masses, or all the planets in the solar system, so that the Sun and Earth move around a common centre of gravity in space, with the result that, according to Newton, "There are as many orbits for a planet as the number of times it orbits." The orbit of Pluto is now said to have been proven to have chaotic potential. Similarly, economists must rid themselves of the misconception that economic interactions and economic policies always lead to similar outcomes cete-ris paribus. Instead, there is a sensitive dependence on initial conditions. The success of earlier Keynesian economic programmes therefore says nothing about their effectiveness in the next current situation.

current situation. For a long time now, Keynesian stabilisation attempts have been accused of only intensifying the cyclicality of the system, for example, through time-lags until the effects become effective. Similarly, the massive expansion of the monetary base to combat the consequences of the financial crisis is currently arousing the fruit that "the devil is being cast out with the Beelzebub".

It would therefore be promising if it were possible to "recapture" chaotic oscillations through very small but targeted and well-timed measures. And - this is indeed a hope shown by recent research. Decisively involved in these theoretical considerations and practical experiments was the already extensively quoted James Yorke, who, together with Robert May, spread the logistic equation in modern science. In 1990, together with Edward Ott and Celso Grebogi at the University of Maryland, he succeeded in proving theoretically that a chaotic system can be stabilised on a periodic trajectory with the help of a control loop that consumes very little energy. The method has become known as the OGY method after the researchers' initials. Even before the paper was published, the first practical application of the principle took place in 1985 - in space. At that time, the space probe ISEE-3 (International Sun-Earth-Explorer) was transformed into the star ship ICE International Cometary Explorer with minimal fuel expenditure by catapulting it from a geostationary orbit through half the solar system to visit the comet Giacobini-Zinner. A >>little push at the right time <<was all it took. Similar successful applications of the OGY principle were demonstrated in electrical engineering and heart surgery. On the road to success, however, a fork in the road soon became apparent: some researchers attempt to create a mathematical model of the application through intensive measurement (in the case of the heartbeat phase space, over 100,000 heartbeats in 24 hours) before venturing into interventions. Others, however, believe that this is not necessary. In 1992/93, for example, a group of researchers at the University of Cailfornia "partially succeeded in resuscitating the arrhythmic beating of a rabbit heart without knowledge of the underlying mathematics." Behind this is a further development of the OGY approach to chaos control. What began as basic research is thus slowly becoming fruitful and practical in application: with the invention of the OGY method in

1990, the number of publications on chaos control is increasing. In 2009, there were already over 1000.



Markus Dahlem, a medical doctor, describes the progress made since then as follows:

"The first method by Ott, Grebogi, and Yorke was presented in 1990 [1] (also called the OGY method). The OGY method is still quite elaborate and simplified I will call it a tightrope walk with eyes wide open.

In 1992, Pyragas found a new method [2]. This one borders on magic. Not only is a target state (a UPO) found blindly, but the control force disappears as soon as the target state is reached. This makes this method minimally invasive, because a correction is only made if a wrong step is taken.

The resolution of the magic lies in self-control, i.e. a closed but time-delayed feedback loop. In this way, Pyragas' method ultimately imitates what nature itself has invented thousands of times before, systems that regulate themselves. The study of these systems in living organisms is of course called - and here another circle closes - physiology.""

And solid-state physicist Thomas Bernhardt wrote in 1995: "In connection with possible applications in technology or nature, reference is often made to the flexibility that the control of chaos could offer (see e.g. OGY90). For since there are an infinite number of UPOs embedded in any chaotic attractor, it should be possible to stabilise the system to a selected UPO. Using the simple example of the diode resonator, this paper has shown that this is possible. Even if, in practice, an infinite number of UPOs would not be achievable, the prospect of being able to switch between different periodic states with small control signals is very tempting. But real applications are still a long way off. So far, almost exclusively physical and chemical model systems have been controlled."

As an established finding of chaos research, it can be stated that the phenomenon of (self-) reflexivity, insofar as this property belongs to a system - that this property causes chaotic behaviour in this system. Chaos control according to Pyrygas indicates that reflexivity, which is brought in by a

meta-level, a system administrator so to speak, can unfold an ordering and stabilising function. In a nutshell, one would like to speak of "white" and "black" magic. "Black" reflectivity would thus be connected with atomistic actions of lower system levels. One example is the introduction of globally networked computer technology on the stock exchanges. In parallel, individual traders introduce their own hardware with ever higher and faster data throughput and ever more sophisticated software programmes. This results in a systemic paradox: those agents who promise themselves information advantages and protection from the observed volatility of stock exchange prices through improved analyses and price forecasts only create greater reflexivity and thus reinforce the chaotic, fractal dimension of the market through their own actions. Stock exchanges thus become super-reactive systems with ever greater volatility. Financial investors are thus faced with a complete dilemma. The investment banker Kathy K. Sato has found a convincing answer to this problem: In her "Theory of Wild Beats" she describes a new investment style of "phase investing", in which investors no longer have to try to predict market developments (or, as Keynes put it, "to beat the gun"). In the future, it will be more important to always invest with the market in those asset classes that correspond to the current market trend, just like a herdsman.

In the logistic equation, all the complexity is condensed in the coefficient r. One could say that r represents the resultant and the desideratum of the expectations of all market agents and thus the amount of information that is fully reflected in prices according to Fama's market efficiency hypothesis. However, this view would again fall behind the insights that have meanwhile been gained into the dynamics of complex market processes. A dynamic theory must also consider the formation of expectations as an economic process. Expectations are not formed in individuals solely as the processing of received information against the background of a framework of knowledge and personal predispositions. With the exception of personal sensory impressions, information must have been produced, i.e. acquired, beforehand. It would seem to me to be a great step forward if one could understand that and in what way information work is done in an information and knowledge society, and that this concrete work is traded as an immaterial economic good on information and knowledge markets. This opens the door to a broad research programme that poses its questions along the classical economic categories of its subject:

Value and price of information

production of data, information, education and knowledge and their conceptual specification for economic contexts

Allocation of knowledge

Distribution of information

Accumulation of data, information, knowledge; problem of "original" accumulation

Explanation of the economic nature of the profit from information processes in comparison with the classical revenues basic rent, wages, profit, interest, rent, quasi-rent (presumption of a new profit category: "progfit" in the sense of the information benefit for the prediction of future data).

Institutional economics of information-producing institutions, e.g. universities.

Institutional economics of information-distributing institutions = economics of education

Economics of economic sectors

General theory of intangible economic goods: value, price, utilisation; investment and financing

Phenomenology: explanation of observed facts, e.g. the historically unique rapid accumulation of capital by companies like Google and Facebook

Theory of the politics of the knowledge economy

Some of these questions are addressed at the George Soros-funded "Institute of New Economic Thinking". Researchers interested in the problems outlined here are recommended to network with this institution.

A very special question also arises directly from the economically interpreted logistic equation:

If the miraculous coefficient r does not represent a perpetual constant, but the result of interaction around information - which components then flow into this quantity and how can they be measured? What are the dimensions of the categories that produce this value and why is r still only a dimensionless absolute number? What is the significance of the limit value 3.57, "behind" which chaos begins?

If we could answer these questions, we could recognise critical system states at an early stage and possibly actually "counteract" them with little effort before problems become manifest. It would probably be important for this that the measurement of the system state is carried out continuously in short intervals, so to speak "in real time", so that the economic-statistical methods are improved in the sense that important key figures are generated online and automatically from the economic process. At the same time, we need to better understand the "mathematics of interaction". It seems to be based, among other things, on the fact that systems/markets have a history or memory. For example, in markets with rising price expectations, buyers need to find at any point in time enough sellers who had bought at more favourable conditions and whose profit expectation (utility) is now satisfied so that they agree to sell their assets to buyers with (even) more optimistic(er) price expectations.

Keynes elaborated the dominance and steering function of the financial markets over the goods and labour markets. It is time to recognise that the economic system we live in is now subject to the dominance and internal contradictions of information markets. Information not only controls the economy, it permeates it. At the same time, the modern know-economy also appears as an immense accumulation of information. This is a historically evolved fact, not a theoretical fiction. The theory, however, goes to the deepest roots of economic as well as scientific thinking when the physicist Carl Friedrich von Weizsäcker writes: "A quantity of information is obviously neither a quantity of matter nor a quantity of energy. ... (In)Form(ation) is not a third thing beside them, but it is their common ground." ... "The abstract structure of quantum theory suggests that information should be understood as the underlying and insofar as the substance. The Ur is then an >>atom of information<<. This is also how the physicist Aton Zeilinger sees it when he says: "To this I would like to make the radical proposal: Reality and information are the same thing."

Finally, von Weizsäcker transforms not only the basic physical quantities but also the economic categories into information: "The thesis is that operational definitions of the terms information and utility can be given, according to which both terms are essentially identical. Accordingly, one could ... conceive utility as a measure of information understood in terms of content."

Parallel to this, "work" can be recognised as the activity of shaping: Shaping, i.e. also information. Abstract labour is information labour. Labour power would thus be a shaping potential force, because, according to Zeilinger: "The world is everything that is the case, and also everything that can be the case." Insofar as labour power is bought and sold, it is therefore always an immaterial economic good. In fact, the economy is fundamentally always an economy of potential. Every effort is an act of hope.

The contradiction that is often seen between the real economy and the financial economy is eliminated in this perspective. What sounds abstract is easy to understand in practical terms: The farmer sows with the prospect of a good harvest, the merchant sends trading ships with trepidation for safe arrival. The factory owner produces in the expectation of sufficient sales and his workers toil with the claim to their wages at the end of the month. At the time of the decision, nothing about these motives is "real". Most economic actions are obviously based on a mediation between the present and the future. A mental anticipation of time. An expectation in the sense of a prognosis, which, as we know, can often fulfil itself. The mental advance in time is not a trivial process. It is based on a fundamental human ability, the imagination. However, imagination does not function solely according to operational rules, it does not follow strict causality. It is not deterministic, but has degrees of freedom (like quark). Imagination allows new things to emerge, and is thus the basis of creativity.

These considerations give clues for two strands of basic economic research:

On the relationship between informAtion and TIME.

On the nature of creativity

In the outlook, let us venture the thesis: the knowledge economy is a process of creation. Increasingly, only those goods that deserve the attribute "creative" have a high value in this form of economy. One cause that produces creativity is "emergence". Emergence is the product of complex adaptive systems.

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APPENDIX 1:

Hicks and Keynes: Squaring the Interest Rate

The quoted passage on the decision rule for liquidity diposition based on the square of the current interest rate is one of the few mathematically formulated statements in Keynes' "General Theory ..." and, if closely examined, even the only such statement. In my opinion, it is therefore astonishing that this statement has not found any attention in the literature of Keynes exegesis - at least as far as I can see. Apparently, later readers did not find the statement significant. Or they puzzled over the knowledge from which Keynes might have derived this rule, because Keynes gives us no indication of how he justifies the connection. In particular, there is no reference to the literature. If one tries to verify the statement by exemplary recalculation, one falls into deep despair, because the statement does not seem to be formulated precisely: different calculation approaches do not confirm the statement.

Thus, three questions arise for this passage, which are central to the course of the argumentation in this essay:

1.) Does the square formula of interest rate expectations have a theoretical basis, where can it be found and is it conclusive?

2.) Is it possible to ascertain whether investors "in Keynes' time" actually used such a rule of thumb as a guide?

3.) Do investors in bond markets today still base their liquidity decisions on a formula in which an analogous interest rate square dominates? Is the interest rate square rule therefore empirically significant?

The first question can be answered reasonably plausibly today (2016), 5 years after the first version of this essay: Keynes' statement is possibly the reproduction of a rule of thumb for capital investment that is well-known in financial circles. The exact justification and derivation of this insight, however, come from J.R. Hicks. The theoretical justification of the square rule of interest can be found in Hicks' work "Value and Capital", which was published three years after Keynes' "General Theory".

As is well known, Keynes and Hicks worked together at Cambridge University during the period when the General Theory was written, but only in 1935, the year in which Hicks received a lectureship at Cambridge University, while Keynes had already completed his General Theory. From the literature available to me, I was unable to ascertain whether Keynes and Hicks had any contact at all during this period, and if so, how intensive it was. The known facts, however, give rise to well-founded assumptions.

Hicks, at any rate, is reported as saying:

"The move to Cambridge in 1935, the year of his marriage to Ursula Webb, marked a

substantial change. The sociable atmosphere of LSE was replaced by one in which, 'people

are terribly prone to quarrelling with each other. At that time the Cambridge faculty was

divided into parties which wouldn't talk to each other. I didn't enjoy that at all' (in Klamer,

loc cit, p. 170) "Klamer, A. (1989) An accountant among economists: conversations with Sir John R. Hicks, Journal of Economic Perspectives, 3, pp. 167-180.LH) "Working much more in isolation than before, the major product was the consolidation of his earlier work into his most famous book, Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory, published in 1939 (second edition 1946)".

Hicks had received a solid - if not outstanding - mathematical and statistical education at Clifton College, Bristol. Among his teachers was the famous Frank Yates, who worked with the biostatistician R.A. Fischer. Fischer, in turn, was associated with US statisticians whose advances made possible the establishment of the National Bureau of Economic Research (NBER) in 1920, whose work with Maculay in 1938 gave rise to the concept of 'duration' for investment in bonds. Hicks studied mathematics at Balliol College, Oxford University, from 1922. There, after one year, his teachers meant that he could hardly make any progress or merit in this subject, whereas laurels could still be earned relatively easily in the academically miserable economics. Hicks therefore changed to the then new course "Political Philosophy and Economics", and spent a postgraduate year at Oxfort

before joining the faculty of the London School of Economics as a "junior member" in 1926. He remained there until 1935, when he was called to Cambridge to lecture. Hicks of-fenced his appointment to the LSE thanks to a protective family network. In any case, he did not even have an economically relevant thesis from Oxfort to show for it. Hicks first appeared as a scientific economist in 1930 with a publication in the Economic Journal: "Edgeworth, Marshall and the Interminateness of Wages. (EJ, 40, pp 215-231)" This may also have brought him to Keynes' attention for the first time, as Keynes had been the editor of the Economic Journal since 1911. In this position, Keynes was the gatekeeper for any academic career in economics, and it is very likely that Hicks sought proximity to Keynes for this reason alone. In retrospect, Hicks' contribution to the Econmic Journal was judged to be insignificant: "However, this consists of a summary of the separate contributions, rather than containing new analysis". Until 1939, Hicks had no further publication in the Economic Journal, but in 1932 and 1934 he attracted attention with publications in "Economia" and at Macmillan. For Keynes, who was then already considered a 'giant' of economics, Hicks as a colleague at Cambridge must have had little to offer, especially since Hicks' mathematical-neoclassical approach was precisely the kind of economics that Keynes wanted to leave behind. From all this we may conclude that Keynes had no special connection with Hicks. Nor did Keynes include a single work by Hicks in the bibliography of General Theory. Conversely, we meet a Mr. Hicks who, immediately after the publication of the General Theory, worked intensively on Keynes and endeavoured either to gain interpretative sovereignty over his work (as is well known in the graphic illustration of the IS-LM curve) or to make Keynes' expositions appear only as a special case of his own theory - not least through his book "Value and Capital". Even if we now enter the realm of speculation, we may assume that during his time at Cambridge Hicks may have tried to ingratiate himself with Keynes and to be of use to him with sweets of his mathematical knowledge.

In retrospect, Brillant Lucy comes to the following comparison of the approaches of Keynes and Hicks: "We begin to study the term structure of interest rates in a context of certainty using "A Treatise on Money" (1930) where the theory initially appeared and show that Keynes and Hicks reach the same conclusion. Then, we will see that it is not the case in a context of uncertainty. First we introduce uncertainty with "The General Theory" (1936) where Keynes refers to a risk of a liquidity loss related at once to a lenders' "disappointment risk" (Keynes). Second, we consider "Value and Capital" (1939) where Hicks refers to borrowers' risk of a rise in spot [short-term] interest rates (Hicks) and to "professional investor`" liquidity risk."

There was little research into the actual behaviour of institutional private capital investors at the time, and little more than rough rules of thumb for their investment strategies in practice . Poi-tras reports:

"The old finance school was a diverse and loosely knit collection of ideas and individuals. ... Because of the focus on analysing characteristics of individual securities, it was not possible to also identify inexorable laws common to all `capital asssts`. Rather, to achieve the objective of identifying-ing securities providing superior investment returns, these texts do provide `rules of thumbs` and `anecdotes` to employ in, say, the analysis of the accounting statements of publicly traded firms."

Keynes first focused on the group of speculative investors when he branded their exaggerated interest rate expectations as one of the three main factors influencing the Great Depression in his "Trea-tise on Money" in 1930. However, Keynes does not give an assessment of the behavioural maxims of these capital investors in the Treatise, but only in the "General Theory", where he presents the square formula discussed here.

Alternatively to the hypothesis of a mental 'borrowing' by Keynes from Hicks, it is conceivable that the square formula was practically established as a rule of thumb for bond investors in the 1920s/1930s. After all, a theoretical derivation - in anticipation of Hicks' explanations - had already been made in 1893 by George James Lidstone. However, his findings may be considered "buried" and were known at best in the world of insurance actuaries, who were isolated from academic circles: Lidstone's work has only recently been rediscovered by financial-historical research. Hicks is therefore undisputedly regarded today as the "discoverer" of the elasticity of interest rates, without the economic community being particularly aware that Keynes had already used this discovery in his "General Theory".

We can therefore conclude: Either Keynes uncritically adopted the square formula as a rule of thumb known to bond investors, which thus apparently needed no justification. Or he received the mathematical proof of its universal validity from a third person whom he deliberately did not want to mention in an appreciative manner. As things stand, this can only have been Hicks.

I hereby reproduce in my own translation the passages in which Hicks justifies the square formula of interest rate expectation in his work "Value and Capital":

<148>"The great advantage of this first model, which we should aim at, is the simplification of the complex system of interest rates existing in practice for different maturities into a uniform interest rate. (Provided one neglects default risks, only one uniform interest rate needs to be considered anyway). When economists discuss interest rate problems, they often talk about the interest rate. It looks as if they have some kind of simplification in mind (like me); but the interest rate in other papers is actually the long-term capital market rate.

Footnote: The interest rate in Mr. Keynes' "General Theory" is the long-term interest rate.

Let us imagine, then, a functioning economic system in which <149>there is as yet no future market for goods and services and only one form of credit. At the beginning of this book, we assumed a short-term economy in which there were only credit relationships with a term of one week. Now let us assume that credit is granted for an infinite period of time. In each system there is only one type of collateralisation at a time. In the short-term, short-lending-interval economy discussed at the beginning, this collateral consists of the promissory note, i.e. the promise to repay a certain sum at the end of the week. Meanwhile, in our new model - the short-term economy with a long lending period - it is an open-ended obligation, namely a promise to pay a certain sum permanently at regular intervals as interest on the loan.

If the only interest rate set up in this market is the rate for loans of infinite duration, then the rate paid in this economy for any given duration is always a matter for foresight. Even the interest rate on one-week loans (the rate that prevailed in our first model) becomes a matter for personal prediction in the short-term economy with long maturities. If someone wants to borrow money for a week, he can only do so in one particular way. He must issue a bond with an infinite maturity at the current interest rate R and then make provision to repay the bond at the end of the week at the then given market price, this market price being determined by the interest rate R` prevailing in the second week. The actual interest rate for a one-week bond thus depends on the issuer's expectation of the future interest rate R`. The present value of the bond will change over the course of the week in proportion to R/R`. Therefore, the effective interest rate to be paid is

$$R + R/R`-1,$$

which is less than R if R` turns out to be greater (>) than R. Thus, the interest rate at which people may expect to borrow or lend for short maturities depends on their anticipation of the future course

of market rates; less than the current interest rate in the case of rising interest rate expectations, greater than the current interest rate if a falling market rate is expected. …"

<p.260>As we saw in Chapter XI, the reciprocal relationship of different interest rates depends partly on risk factors and partly on the expected path of interest rates in the future. ... Let us now develop our argument for the assumption that interest rate expectations are directed towards the expectation of short-term interest rates. ... <p.261>If these expectations are inelastic, the current long-term interest rate cannot fall by more than a very small amount.<p.261>If, for example, the current long-term interest rate is 4%, and it is also assumed that it will also be 4% at the end of the year, then 4% is also the return that can be earned by investing money now, in contrast to the current cash position and later investment in bonds at the end of the year. If, however, the expected interest rate remains at 4%, but today's interest rate falls to 3 7/8 %, then the net amount that can be earned by a one-year bond (taking into account the expected capital loss) is only 3/4 %. If today's interest rate falls even a little further, the net return on the one-year bond becomes negative. Thus, if one takes into account the riskiness of investing in long-term fixed-income securities 1, (HICKS footnote 1: compare Chapter XI above -INTEREST, pp. 141-152, LH), it becomes clear that even a very small fall in the long-term interest rate is enough to discourage people from buying bonds, at least as long as they have the impression that the fall is only short-lived and that the interest rate will soon return to its original level. 2

HICKS footnote 2: Since (as explained on page 149) the net return achievable by long-term investment in a given period is R +(R/R`)-1 (where R is today's long-term interest rate and R` is the interest rate prevailing at the end of the period), it is easy to calculate the maximum possible decline in the interest rate. Because R+R/R`-1 must be greater (>) 0, so R must be greater (>) than R`/(1+R`); approximately R greater (>) R`(1-R`).

we get the "devil's polymer" with R >(R`-R`2) or 1> 1/R (R`-R`2)(LH).

So if the year-end rate is expected to be 4%, then today's rate cannot fall by more than 4% from 4%; and so on. This is the maximum fall under all possible circumstances; because it neglects risk, it exaggerates the practically possible fall (even still; LH). Compare Keynes, General Theory, page 202."

(emphasis added by LH).

So the reference to Keynes' quadratic equation of interest is clear. Or, to put it another way, there is every indication that Keynes borrowed intellectually from Hicks, which he has concealed from the reader. Nevertheless, Keynes can hardly be blamed for this theft, since Hicks had only published his considerations after the General Theory.

In any case, this answers the first two questions posed at the beginning:

The validity of the interest square formula is mathematically proven to be generally valid, although it was most likely known and used in practice by speculative bond investors in Keynes' time as a mathematical rule of thumb. The devil's polymer of interest thus underlies at least the actions of financial market actors in the economic reality that existed before the Second World War.

But what is the actual meaning of the function a(x- x2) today?

Superficially, the answer is: none!

All practical textbooks for financial market experts today begin with an explanation of the mathematically defined technical term "duration". This ratio was first introduced in 1938 (almost at the same time as Hicks and Keynes, but demonstrably independent of the two Britons) by the Canadian-born US statistician Frederick R. Macaulay. A term of the above kind is not found in the usual presentations of duration algorithms.

Fredericks R. Macaulay: Both his grandfather, Robertson Macaulay (1833-1915), and his father were important figures in Montreal business and society.3 His grandfather served as the second President of the Sun Life Assurance Company of Canada from 1889-1906, resigning the position in favour of his son, T.B. Macaulay who served as the thirdpresident of Sun Life from 1906-34.

Significantly, alumni records at the University of Colorado indicate that in 1921, (Frederick) was practicing as an attorney at law in Berkeley, California with offices at 2442 Hiyard Avenue. It was during his time at Berkeley that Macaulay made the connection to W.C. Mitchell that was to have such an important impact on his future endeavors. Though Mitchell had left for a position at Columbia prior to Macaulay's arrival at UC, his complementary research agenda and the academic network led Macaulay to decide to pursue a PhD under Mitchell at Columbia. Based on his published contribution to the first major research project undertaken at the newly formed NBER National Bureau of Economic Research (Mitchell 1921-2), in 1924 Columbia granted Macaulay a PhD in Economics. Upon arriving at Columbia, Macaulay was able to secure a position on the research staff of the NBER, a position that he held until the completion in 1938 of his special study on the cyclical behaviour of interest rates. Because none of the staff of the NBER was paid more than a modest stipend, with senior staff being employed on a part-time basis, it was expected that the bulk of staff income would come from university teaching positions (Fabricant 1984, p.31). To this end, from 1921-26, Macaulay lectured at the New School for Social Research. His area of expertise combined with a growing network of contacts in the financial markets led Macaulay progressively into the business of financial consulting.

It is Wesley Mitchell that produced Business Cycles (Mitchell 1913) a book which Arthur Burns (1952,p.22) describes as "one of the masterpieces in the world's economic literature".

Wesley C. Mitchell joined the faculty of Columbia University in 1913. Except for a brief peri-od of government service at the end of WWI and three years as a lecturer at the New School for Social Research (1919-1921), Mitchell was a member of the faculty at Columbia until his retirement in 1944. It was during the New School period that Mitchell was instrumental in organizing the NBER, where he served as Director of Research until he resigned in 1945. From the founding of the NBER, "the National Bureau was the focus of his intellectual inter-est, the emotional centre of his own work, and the work responsibility that lay closest to his inner life" (Burns 1952, p.102). The NBER was established with grants totaling $24,000 with which Mitchell was able to hire a small research staff to undertake the first major study on the size, growth, fluctuation and distribution of national income. The initial research staff for the national income study had three members: Willford King, Oswald Knauth and Freder-ick Macaulay. Although the published results of this study (Mitchell 1921-2) appeared within three years, there were a number of follow-on business cycle projects generated by

this initial effort. Among these special studies that got underway in the early 1920's was one on the cyclical fluctuations in interest rates undertaken by Macaulay."

However, interpretations of the meaning of this ratio and explanations of its usefulness for the practical actions of investors seem to be as varied as modern accounts of quantum theory. In addition, the concept was further developed in later years, so that ratios such as "modified duration" and "effective duration" were added, until it finally became (by Frank M. Redington, 1952) the strategy of "interest rate immunisation", with which investors can supposedly hedge against interest rate risks. In the meantime, the "dollar duration" is also used as an important indicator for the total return of a reinvesting fixed-interest investment under changed interest rate expectations. Hicks' contribution was forgotten by financial market practitioners, although his view of "interest rate elasticity" was just another angle on the duration problem. The mathematical content of "duration" and "interest elasticity" is therefore ultimately identical, but the common textbook presentation undercuts the Hicks version.

"While Macaulay, the institutionalist, introduced duration to empirically measure the length of a bond's sequence of cash flows, the 'revivalist' neo-classical economist John Hicks (1904-1989) de-developed the duration concept to theoretically measure the elasticity of the "capital value of a stream of payments" with respect to a change in the discount factor $ = (1 / (1 + y)). Footnote 8.

Unlike Macaulay (1938) where the cash flows of a straight bond are used to define the cash flows (CF = C or C + M), Hicks permits the individual terms in the stream of cash flows (CFt) to vary across time periods (see - Appendix). Instead of referring to the 'duration' of the cash flows, Hicks uses the terminology "average period". Like Macaulay, Hicks provides a solution for the average period of a fixed payment perpetual cash flow. While Hicks (1939) was widely read at the time, due both to the academic stature of the author and the importance of the content to neoclassical theory, the 'capital value' context of the Hicksian duration measure was sufficiently obscure to financial appli-cations that, in demonstrating the elasticity property of Macaulay duration, Fisher (1966) did not reference the contribution by Hicks (1939).-Footnote 9

Footnote 9:

As Weil (1973, p.590) <Weil, R. (1973), "Macaulay's Duration: An Appreciation", Journal of Business 46: 589-92> observes, this does not mean that Hicks was completely unrecognized. Because credits the relatively obscure Grove (1966) <Grove, M. (1966), "A Model of the Maturity Profile of the Balance Sheet", Metroeconomica 18: 40-55.> with having "been the first to cite both Macau-lay and Hicks." However, prior to Weil (1973), mainstream sources that employed the duration measure to analyse securities, such as Durand (1957), did not recognise Hicks (1939), though a vari-ety of other sources, such as Redington (1952), were identified."

Appendix: Derivation of Duration as an Elasticity

Recognition that Macaulay duration can be interpreted as a point elasticity measure of bond price change with respect to a change in interest rates was sufficiently 'new' that Hopewell and Kauf-man (1973; Hopewell, M. and G. Kaufman (1973), "Bond Price Volatility and Term to Maturity: A Generalized Respecification", American Economic Review: 749-53.) provide the derivation of the elasticity representation in full.

More precisely, elementary economics teaches that using the unadjusted derivative to measure the sensitivity of a variable Y, say quantity demanded, with respect to the change in another variable X, say the price of the commodity or income of the consumer, is ineffective because the measurement

unit dependent starting levels of X and Y will impact the result. Instead, the elasticity measure uses the changes in the derivative scaled by the initial level of X and Y. Where appropriate, a minus sign is added to ensure that the elasticity is a positive number."

At this point, the suspicion creeps in that the aforementioned devil's algorithm no longer appears in the modern equations for determining "duration" because, for reasons of "elegance", in the course of mathematical transformations, a minus sign in the initial equation has simply been defined away. In addition, as Henderson points out, the duration ratio is merely a linear approximation of the convex curve of the relationship between market price and interest yield: "In mathematical terms, convexity is the second derivative of the price-yield relationship.

Whether the second derivative of a chaos-inclined function also tends towards chaotic changes may still await general mathematical proof - I am simply not aware of this. However, a hint given to me by Lothar Krätzig-Ahlert, a mathematically competent scientist, says that also completely different "harmless" looking types of equations can lead to "chaotic" solution processes. Accordingly, it would have to be investigated whether in particular the modern mathematical rules used for "interest rate immunisation" belong to the category of chaotic equations.

E-mail from Mr. Lothar Krätzig-Ahlert dated 17.03.2020 to Leander Hollweg:

"I have now understood your question: Are there situations in which the duration shows chaotic behaviour? This is in itself a purely mathematical problem, which in my opinion has nothing to do with a positive or negative sign within the differential equation, which one could then solve iteratively. You would then already notice whether chaos occurs or not. The father of chaos theory, Edward Lorenz, also proceeded with trial and error. It has something directly to do with the differential equation as such, which describes the duration. In Robert May's publication of July-August 1976 on "Bifurcations and Dynamic Complexity in Simple Ecological Models", The American Naturalists, Vol 110, No. 974, a number of equations are listed in the table on page 575 that can become chaotic. So there are also equations that do not contain a minus sign. The corresponding page is attached. The preconditions for chaos are listed one page earlier, on p. 574 below. So you would have to check the original differential equation to see if the conditions are met, or solve the duration iteratively and then run the "growth parameter" and see what happens. In my opinion, this would be the simplest solution. Which duration equation should be investigated?"

In the above "Appendix" by Poitras, there is a reference to the mathematical derivation by Hopewell & Kaufman. In this paper the authors lament "The apparent inability of ana-lysts to explain ( ) unusual bond price patterns reflects an incomplete understanding of the math-ematics of bond prices." Because: "...the volatility of any conventional highgrade bond results from the interaction of three factors: maturity, coupon and the starting level of yields".

The dependence of a system's course on its initial values is a typical characteristic of chaos-prone relationships. It should be noted that we are dealing with a system with three interdependent influencing factors, the independent variable of which is the starting level.

Hopewell and Kaufman state: "The relationship between duration and maturity is non-linear and complex. ... and: "As the relationship between maturity and duration is complex, the relationship between maturity and bond price volatility is also complex." Finally, the authors point to other

factors that influence an interest-rate-dependent investment strategy, citing pre- and post-tax returns and default risk. T. M. Henderson focuses above all on the time yield curve as a further influencing factor and comes to the comparable conclusion:

"Duration is a summary measure of maturity, coupon and yield effects that is used to ap-proximate risk. ... Particularly in a volatile interest rate environment, duration is an ina-dequate measure of interest rate risk."

Henderson gives a practical numerical example in which, in the case of seemingly identical bonds, depending on the market situation, completely opposite performance can occur.

I (the author) would like to point out that Hopewell/Kaufman do not mention the real yield, i.e. the inflation rate measured at the time of the determination of the interest rate for the recent past and the expected inflation rate or their relationship to each other. My assumption is that it is this variable that makes up the unknown starting parameter "r" or "a" in the devil's polymer r (xn - xn2) or in the corresponding Keynes model world a (in - in2). This is supported not least by the fact that the inflation rate is represented dimensionlessly as a pure (decimal) number.

Uli Deker and Harry Thomas described a system with three variables that depended on the initial conditions in the magazine "Bild der Wissenschaft" in 1983, when the chaos theory was still quite unknown even among scientists. I reproduce its description here in full:

"Under what harmless circumstances chaotic motion can occur is demonstrated by a fictitious planetary system in the centre of which there are two suns of equal weight instead of one. They run on elliptical orbits around their common centre of gravity. On the line perpendicular to the orbital plane and through the centre of gravity, a single, very light planet moves. Unless it has enough energy to escape from the system, it is attracted by the suns, passes through their orbital plane at great speed, is slowed down and then brought back again.

This game is now repeated, the planet constantly and completely regularly oscillates through the sun's plane - or so it is thought. It is also true if the sun's orbits are exactly circular. But as soon as they are even slightly elliptical, the behaviour of the planet changes drastically. It now depends sensitively on its starting position and speed.

For example, the times at which the planet passes the solar plane are interesting. In a regular motion, these times follow each other at constant intervals, say every tenth day or once a year. The inhabitants of the planet would align their calendar accordingly.

In the case of chaotic movement, the sequence of days on which the planet passes through the solar plane is extremely sensitive to the planet's starting conditions. It can even be mathematically proven that one can specify any sequence of days and have the planet start in such a way that it passes through the solar plane exactly on these days and never otherwise. Arbitrary means really arbitrary, for example every day for a week, then not at all for a millennium, then again on every first Saturday of a month or on all days whose da-tum contains a prime number. All you have to do is "set" the position and speed of the planet at the beginning. The rest is taken care of by the gravitational forces, as Newton already described them. Would the inhabitants of this planet develop a causal view of the world?

This example makes it clear why we speak of "chaos". Characteristic is

- the sensitive dependence on the starting conditions ...

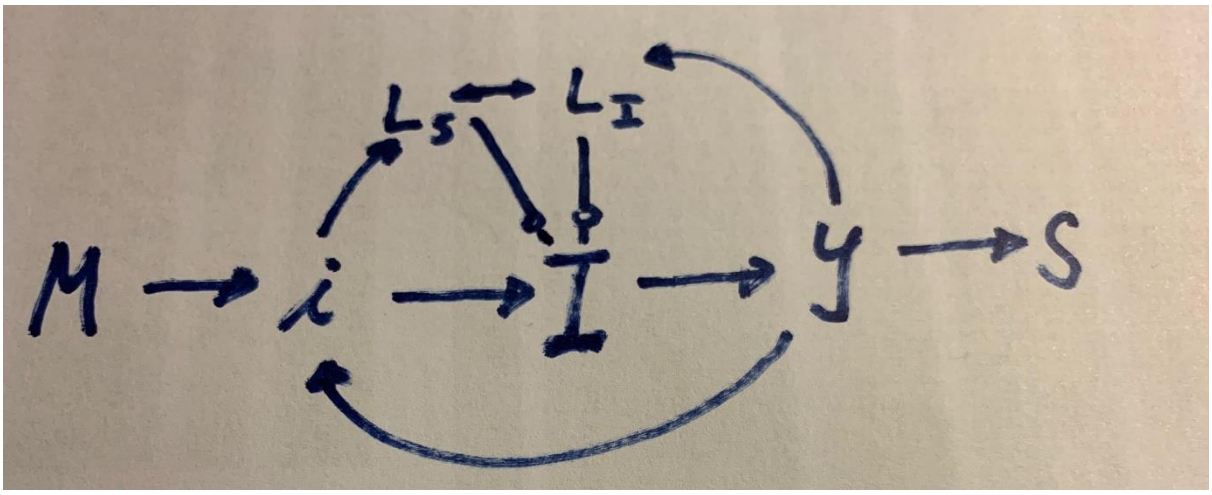
- the resulting violation of the strong principle of causality: similar causes no longer have similar effects

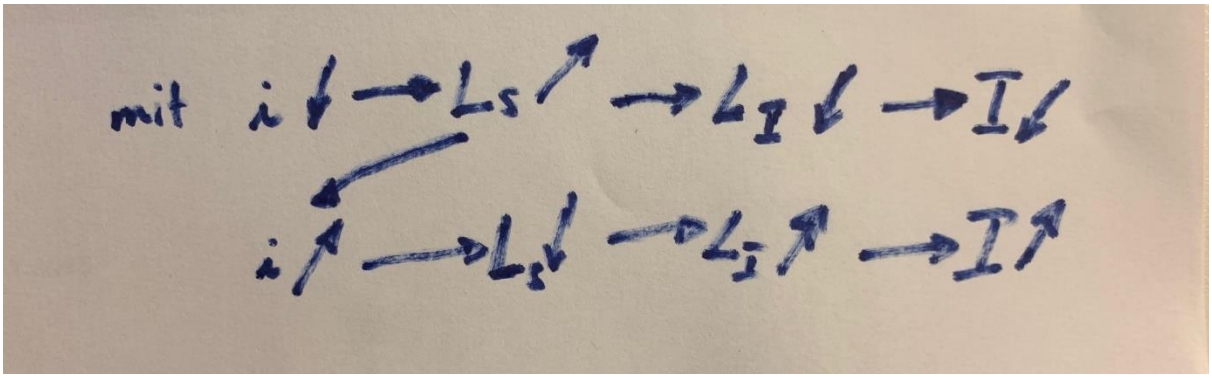
- the irregularity of the movement, it does not follow a simple pattern ...

- the incalculability of long-term behaviour: in principle, it is impossible to determine the starting conditions with complete accuracy. Movements from neighbouring points are similar ( ) only during an initial period. Calculations are still useful for this long period. After that, the uncertainties have increased so much that the prediction is only: The planet is somewhere. How long the initial period lasts varies greatly from system to system. It also depends on how precisely the starting conditions are known.

Chaotic motion is not only found in mechanical systems. …. Mathematically, all systems that have more than two degrees of freedom are highly suspect. This is also true of all complicated models of economic, social or ecological processes."

In Keynes' macro-economic analysis of the economic process there is such a "suspicious" indeterminacy, which I already noticed as an undergraduate student: because Keynes describes the interest rate as a function of the demand for liquidity and, conversely, the demand for liquidity as a function of the interest rate, the effects of an economic stimulus are unpredictable. This becomes clear, at any rate, if one does not imagine macroeconomic development as a simultaneous determination in the form of a Hicks-Hansen-IS-LM equilibrium, but as a gradual process that is triggered, for example, by an expansion of the money supply.





M = Money supply i = Interest rate I = Investment sums Y = National income/production volume

S = Savings sum Ls = Liquidity for speculative purposes Li = Liquidity for transaction purposes

What is interesting about this Keynes model is the effect of an actual interest rate cut (brought about by an increase in the money supply), which - contrary to what mainstream economics assumes - has a dampening effect on investment activity: The interest rate cut causes investors to hold back on buying more fixed-income securities, i.e. to hold more cash. According to Keynes, this reduces the amount of liquid money available for transaction purposes and thus hinders further investment activity. At the same time, however, there is a feedback effect of the increased speculative cash on the interest rate, which is defined as the "price for giving up liquidity". Thus, the tendency of interest rates to rise again sets in motion an opposite effect mechanism. From the constantly opposing interdependencies, it is no longer possible to derive any prediction about the actual or even probable course of investment activity or economic development.

Accordingly, investors are left confused in the "real" world. Their practical woes are well described by Tamara Mast Henderson from a theoretical and empirical perspective:

"The purpose of this book is to bridge the different worlds of theoretical models and practical

market experience, while at the same time to offer an interdisciplinary framework for fixed

income investing and trading.“

The book develops its arguments by increasingly comparing the bond trader's activity to the perilous fate of a soldier on the battlefield and concludes with the advice: "Long-term survival - both on the battlefield and on the trading floor - takes discipline, endurance and courage."

Keynes' simple square rule of interest rates is therefore no longer suitable in the immediate sense as a guiding principle for investors in bond markets. However, as a mechanism of interest rate elasticity, it plays the basic melody over which the increasingly complex influencing factors rise to form an ever more demanding symphony. A conversation I had with Mr Uwe Günther, Managing Director of BPM Berlin Portfolio Management GmbH, on 18 January 2018 reflects this. BPM is the largest private asset manager in Berlin and is predominantly involved in bond strategies.

I record the following statements by Mr Günther as quotations of the spirit:

"I also deal with offbeat, speculative bond strategies, but I have never heard of the quad-rate rule."

"If the rule were true, it could probably be tested by comparing spot and futures markets; but that data does not seem to exist yet."

"Orientation to liquidity may apply to the operational entrepreneur, but not to the asset manager. For us, the mandates given are decisive (in the sense that we should generate an optimal return on a given capital)."

"Hedging strategies for this require that you can separate the credit risk and the interest rate risk of a bond. That is no longer possible today. Market sitation has become multicomplex, with cross-currency and commodity influences to consider."

"We are experiencing globalisation in its purest form plus ECB free-riding."

"People have tried to map market developments with algorithms that worked for decades, but no sooner were they implemented than they suddenly failed."

"I doubt that the square rule has any relevance for our bond strategists. They will see dealing with it as a waste of time. Only our economic analyst may see it somewhat differently."

Appendix 2: Keynes as a Capitalist: Bonds or stocks?

In this appendix I am concerned with the question of whether Keynes had knowledge of the mathematical work of Alfred James Lotka. A positive answer would strengthen the thesis put forward in this paper.

Lotka was an Austrian-American chemist (rather physicist, LH), actuary and demographer who, from 1924, worked for the US insurance company Metro-politan Life Insurance Company, which has since become the largest insurance company in the world.

Lotka's main work dates from "1925: Elements of Physical Biology (the 1956 reprint was titled Elements of Mathematical Biology). With this work, Lotka wanted to launch a new branch of knowledge - physical biology - which consisted of transferring physical principles to biological systems. The basic assumption was that all developments (including 'evolution') can be described by the theorems of thermodynamics as a transformation of energy. In such a model, the entire inanimate as well as animate nature appears as a huge energy conversion system."

Lotka later published important theoretical works on actuarial mathematics, e.g. The Money Value of a Man (together with Louis I. Dublin) in 1930 for calculating the net life income lost through death or disability.

MetLife, as it is known today, was converted from a stock corporation into a mutual company in 1915. According to Robert Leckachman, the "National Mutual Insu-rance Company", of which Keynes was chairman of the board from 1921, also had this legal form. "For a time he was on the board of the Independent Investment Company and later of the Provincial Insu-rance Company. He had a happy hand. Robinson stated: >There can be no doubt about his flair for proper investments.<"

Various recent publications cast some doubt on this statement of the "lucky hand". It is becoming increasingly clear from these publications that Keynes was a very active capital investor and speculator.

What is known about Keynes' activities in the insurance industry? What can we learn about the

National Mutual Insurance Company? Finding this out is not so easy. A search on the internet first leads to the following lead:

"National Mutual Insurance Co. operates as a subsidiary of Celina Insurance Group."

https://www.bloomberg.com/research/stocks/private/snapshot.asp?privcapId=26938751

Celina Insurance Group was founded in 1914 in the US state of Ohio as "The National Mutual Auto-mobile Association" and offered insurance against the risk of an automobile catching fire. Later renamed Celina Mutual Casualty Company, the company experienced strong growth in other

insurance lines in the mid-1920s. In general, the post-war years between 1920 and 1930 were boom years for the insurance business in the USA, especially for life insurance. The National Mutual Insurance Co. is today listed as one of four companies belonging to the group (http://www2.celinainsurance.com/our-history) and is the legal successor of the National Mutual Automobile Association as a non-profit organisation. I was unable to find any further information on the history of the National Mutual on the Internet. There is a risk of confusion with the Nationwide Mutual Insurance Company from Columbus, Ohio or "The National Mutual Life Association of Australasia", founded in 1869, which was often referred to simply as National Mutual Life Insurance, or the -also American- "National Life Insu-rance Company" or a whole series of similarly named companies, which were, however, all founded well after 1921, see https://opencorporates.com/companies/us_mo/I00000662, among others. If National Mutual was already a US company in the 1920s, this would also support the assumption of a connection to MetLife.

In any case, it can be assumed that a US mutual insurance company was a member of NAMIC, an industry association that was very active in 1920 and whose task also consisted of exchanging insurance knowledge. You can find more information on this on the Internet: https://en.m.wikipedia.org/wiki/National_Association_of_Mutual_Insurance_Companies

Elsewhere, the name of the company of which Keynes was a member of the board of directors is given as "National Mutual Life Insurance Company". https://www.maynardkeynes.org/insurance-industry-investing.html . A corresponding search for this company name also tends to land one in the above-mentioned jumble of names of predominantly American companies.

However, it is probably a purely British company. The FAZ journalist Gerald Braunberger noted

"Keynes joined the board of the National Mutual Life Assurance Company in 1919. "Instead of local obscurity, the company thereafter enjoyed national prominence," wrote business journalist Nicholas Davenport. Two years later Keynes took over as chairman, despite somewhat strange ideas: "A life insurance company should have only one form of investment, but should change it every week." The company did not follow this advice, but took up Keynes' idea of also buying shares, which was revolutionary. Usually insurance companies bought bonds or mortgages at that time. Keynes remained chairman of the board until 1938."

According to this, the company was not called "...Insurance", but "...Assurance".

But this information is also incorrect.

Keynes actually held his offices in the "National Mutual Life Assurance Society": "The company was founded in 1896 and is based in Hitchin, United Kingdom. As of September 27, 2001, National Mutual Life Assurance Society operates as a subsidiary of General Electric Capital Corporation" and is today a British pension fund with an outdated management. https://www.bloomberg.com/research/stocks/private/snapshot.asp?privcapId=946822

The evidence for this is in a British archive, the Guildhall Library Manuscripts Section. Keynes' correspondence with this company is kept there, and from there interested economic historians may research further.

Insert: *https://www.history.ac.uk/gh/keynes.htm*

"Guildhall Library Manuscripts Section : John Maynard Keynes

Papers giving a fascinating insight into the economist John Maynard Keynes have recently been acquired by the Manuscripts Section.

The papers form part of the archive of National Mutual Life Assurance Society, of which Keynes was a director from 1919-1938 (he was also Chairman of the Board from 1921-38). They include extensive correspondence featuring not only the affairs of National Mutual, but also his ideas and views on more general economic issues as well as some more personal corre-spondence. They also contain copies of his speeches to the National Mutual AGMs, which became important events in the economic and City calendar.

The papers were presented to the Manuscripts Section, having previously been stored in the National Mutual Assurance Society offices at Hitchin, Hertfordshire. The collection has been catalogued as Mss 34401-584 and is available for consultation, although 24 hours notice is required for access. Our catalogue is available on the City of London Corporation's internet pages (Address: http://librarycatalogue.cityoflondon.gov.uk/www-bin/www_talis).

Keynes' letters are available in the series of directors' correspondence files (Guildhall Library Ms 34486/5). Please note: the Keynes' papers require proof of identity, something which gives name, address and signature."

Researchers may wish to note the following notice that the Guildhorn Libray has now closed and partially relocated: https://www.history.ac.uk/gh/

"Guildhall Library Manuscripts Section has closed permanently and merged with London Met-ropolitan Archives.

Following the reorganisation of Guildhall Library and City archive services generally, the Manuscripts Section of Guildhall Library has now closed. Most of the archive and manuscript collections and archive services for the City of London are now concentrated at London Met-ropolitan Archives (LMA) in Clerkenwell, a mile north east of Guildhall Library. At LMA there is the full range of consultation facilities, conservation support, reprographics services and spacious areas for research and study. The vast majority of archives will now be consult-ed at LMA. LMA's archive catalogue is available online. If you require any further infor-mation please contact LMA by email or phone us on 020 7332 3820.

A few important archives will continue to be housed and consulted at Guildhall Library. The key collections are:

- The Stock Exchange archives and printed collections

- Lloyd's of London archives and printed collections (except the Lloyd's Captain's Registers which are consulted at LMA)

- The City of London Livery Company archives (and some associated collections)

Full information for accessing these three collections is available on our website."

End of insertion

So it now seems less likely that Keynes was familiar with American developments. As a British diplomat, however, he had already had intensive contacts with the USA during the First World War. As a British diplomat, however, he already had intensive connections to the USA during the First World War. In June 1919, he was allegedly offered a position on the board of a foreign bank (unnamed in my source and others). In May and June 1931, Keynes stayed in the USA for extensive studies of the US economy.

By April 1934 at the latest, his comments on the US economy already had a lasting influence on the policy of the US Federal Reserve. In mid-1934, Keynes was again in the US to study the stock and bond markets "for personal purposes". In that year, Keynes completed his first manuscript of the "General Theory" and sent it to numerous recipients with a request for comment.

In The Origins of Asset Management from 1700 to 1960 it reads:

"Keynes, a worldfamous economist, was a prolific and skilled asset manager involved in a range of different investing activities. As chairman of the National Mutual, a life office, he was an investment visionary: he provided insurance companies with an investment blueprint for their future success and he was a successful, innovative investor despite occasional diffi culties after the 1929 Crash. Keynes, helped by Harold Raynes (Actuary at Legal & General), beneficially influenced the path of asset management after 1919."

And now, attention: "The group (Legal & General, LH) expanded in the UK and soon began to acquire overseas life assurance companies, purchasing a pensions business from the Metro-politan Life Assurance Company of New York in the 1930s."[4] Source: <Wikipedi, Legal & General>

And as I reported above, Alfred James Lotka had been working for the very same Metropolitan Life Assurance Company of New York since 1924.

As a mutual company, Metlife was also involved in the information exchange of the cooperative insurance sector, of which the company Keynes supervised was also a member. In the USA, this was - as already mentioned - the organisation NAMIC, which certainly also had connections to the British Association of Mutual Insurance Companies.

There is therefore a distinct possibility that Keynes had early knowledge of Lotka's work.

Finally, I would like to point out the following fact, which is presented to us in the following source as an "Unsual Fact" about Keynes:

https://www.telegraph.co.uk/money/f-and-c-investment-trusts/john-maynard-keynes/ :

9 He took a philosopher on his honeymoon.

In an unusual move, Keynes invited the philosopher Ludwig Wittgenstein to stay while on his six-day honeymoon in Sussex. According to biographers, Wittgenstein made his wife cry.[9]

(9)Then, on their honeymoon, Maynard, with a rare lack of consideration, had invited guests to stay, including the philosopher Wittgenstein:

He used the whole of his six-day visit to exhibit his most antisocial traits. He dominated the conversation ... and Lydia he treated with unconcealed contempt (= undisguised Gerimg-schätzung, LH). When she ventured a mild remark about the beauty of a tree, he crushed her with the blistering epistemological challenge, 'What do you mean?' Lydia, who was already unnerved by the philosopher's incomprehensible monologues, burst into tears, and even Maynard was appalled by his behaviour.

The article goes on to show that Keynes was greatly inspired by his wife's Russian poetic way of thinking and that his strictly logical-analytical thinking changed as a result.

Interestingly, Prof. Dr. (emeritus) Fritz Gruendger commented when I spoke to him at the 2019 Keynes Society meeting about the desperate verbalisation of a mathe-matic statement through Keynes` convoluted formulations "Then Keynes would have been like Wittensteien, "what you can`t say anything about, you have to keep quiet".

An important source of recent research (2018) is the essay by C. Cristiano and M. C. Marcuzzo, already noted, which I include as an appendix in full text.

After reading various sources on Keynes' investment strategy, it is clear that he started investing occasionally in equities early in his life and, when he began to receive a correspondingly high income in 1921/22, he turned mainly to investing in currencies and commodities. Investments in fixed-interest securities are only attributed to him as "unsystematic excursions". In the 1920s, Keynes developed an investment style that was oriented towards the course of the economic cycle:

"An early experiment emerging (...) is to be seen in the 'credit cycle' strategy elaborated in collaboration with Falk and explained in the prospectus of the Independent Investment Company of January 1924 (CWK XII, p. 33). This strategy started from the presupposition that fluc-tuations in the relative values of ...securities generally and of ordinary shares are all affected by a periodic credit cycle. Changes in the short-period rate of interest affect the value of long-dated securities to a greater degree than should strictly be the case, with the result that consid-erable profits can be made by changing from one class to another at the appropriate phases of the credit cycle."

Keynes thus revealed insights that could only have been accessible to a theoretical economist through Maculay's derivation of duration in 1938:

"It should now be clear that bonds with smaller coupons and longer maturities are more sensitive to a given change in interest rates." And in general: "The sensitivity of a bond price to changes in interest rates varies depending on the level of interest rates.

Moreover, one can assume that Keynes - according to his investment style - knew Mitchell's work on the business cycle. From there, connections to the NBER and thus also to the person of Macaulay and his early studies on the development of interest rates are obvious, or at least possible.

As early as January 1928, Keynes explicitly advised against bond investments: "the cen-tre of gravity of business, and therefore of investment, is not where it was [before the war]', and that sticking to the orthodox and restricted range of fixed interest securities would mean 'living in a backwater'.

However, it is much more correct that even in our present day, the capital markets are primarily characterised by investments in fixed-interest securities:

"As of 2017, the size of the worldwide bond market (total debt outstanding) is estimated at $100.13 trillion, according to Securities Industry and Financial Markets Association (SIFMA). The bond market is part of the credit market, with bank loans forming the other main component." ... "The bond market has largely been dominated by the United States, which accounts for about 39% of the market."

"As measured at year-end, the U.S. bond market has been bigger than the U.S. stock market in 24 of the last 25 years. The exception was 1999, when stocks were rather pricey. On average, the market of investment bonds has been 79% larger than the stock market over the last 25 years.13.02.2020" .... The market volume (nominal values) on the German bond market amounted to 3.1 trillion euros in 2014, while the stock market had a volume of 1.5 trillion euros. This makes the bond market twice as big as the stock market."

Quod erat demonstrandum:

1.) Fixed-income securities and thus the trading strategies of the players still dominate the capital markets today.

2.) With regard to duration as the most common indicator for trading strategies in the fixed-income market in modern portfolio management, the following applies: "The investment game is a complex one.

3.) Keynes understood this complexity and recognised it as a fundamental systemic uncertainty factor for economic development.

4.) There is a distinct possibility, even probability, that Keynes was aware of the work of Lotka and Macaulay.

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Abstract

Keynes the investor has recently attracted the attention of several scholars and quite a few articles have come out in the last six years. A description of Keynes's dealings has emerged, assessing his performance as an investor as superior but not as stellar as had previously been believed. However, overall evaluation of Keynes's performance is still lacking. This paper contributes to this growing literature by filling some of the gaps, especially in relation to Keynes's investment philosophy and economic theory, and by undertaking a more compre-hensive review of the available evidence, drawing on some unpublished sources which have not yet been fully exploited.

John Maynard Keynes: the economist as investor*

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1 INTRODUCTION

The title of the second volume of Skidelsky's biography, The Economist as Saviour (Skidelsky 1992), conveys the idea of Keynes as both a guardian and a defender of capitalism. There are, however, other aspects to Keynes which suggest a different depiction, or at least a different perspective. Such is the case of his activity as investor and speculator, which was a constant concern throughout his life.

Keynes started up as an occasional investor in the stock market when he was very young. After 1919, thanks to the proceeds of the best-selling The Economic Consequences of the Peace (Keynes CWK II), his dealings grew in magnitude as well as scope. Currency speculation became the main business for a while, leading to a serious loss in 1920, which Keynes had already helped to recover by 1922, thanks to the revenues coming from speculation in commodities (metals and cotton) and some still unsystematic forays into securities. Meanwhile, investment had become his main source of income. The data on income by source reported by Moggridge (CWK XII, p. 12, table 4) show that commodity speculation took the lion's share during the 1920s -a pattern that probably began to change when Keynes's second major set-back came in 1928, and then in the wake of the 1929 crash. Even though Keynes

went on trading commodities until the closure of these markets in 1939, early in the 1930s he shifted to equities, his main sources of income being capital gains and dividends. Connected to this is Keynes's exposure on the American stock market, which shows up in the 1920s data and then in 1932-1933, before taking on substantial proportions from 1934. Keynes continued to invest until the time of his death in 1946, building up a conspicuous fortune over a period of about a quarter of a century.

Parallel to his personal investment activities, there was an intense career as an institutional investor. Keynes became director of the National Mutual Life Insurance Company in 1919, and then chairman in 1921, a post he retained until October 1938. He joined the board of the Provincial Insurance Company in 1923, lessening his involvement in the board only when he joined the Treasury in 1940. Keynes also entered onto the boards of a group of investment trusts founded by O.T. Falk, a former colleague of Keynes's at the Treasury. He was a director of the Independent Investment Company (1923-1946), the A.D. Investment Trust (1921-1927) and the P.R. Finance Company (1924-1936, Chairman 1932-1936). In addition to these investment companies, there was the Syndicate that Keynes and Falk created for their speculation in foreign exchange, and into which they channelled additional money from friends and relatives (CWK XII, pp. 1, 3-5, 30-35). In 1921, Keynes became Second Bursar of King's College, Cambridge, and then First Bursar in 1924, a post he retained until the end of his life.

Keynes' investor has not been investigated as much as other aspects of his life and work, especially because of the nature of his work.

and work, especially because of the nature of the sources; in his papers there are many files of ledgers, correspondence with brokers and consultants, and accounts which are not always easy to decipher and make use of. Moreover, reconstructing the workings of the financial markets in the interwar period would require a full knowledge of data (asset prices, derivatives, commissions, institutional arrangements) which are not always available.

Notwithstanding these difficulties, Keynes the investor has recently attracted the attention of several scholars and quite a few articles have come out in the last six years (Fantacci et al. 2010; 2012; Holder and Kent 2011; Boyle et al. 2012; Marcuzzo 2012; Chambers and Dimson 2013; 2015; Wasik 2013; Woods 2013; Cristiano and

Naldi 2014; Chambers et al. 2015a; 2015b; Accominotti and Chambers 2016; Chambers and Kabiri 2016; Marcuzzo and Sanfilippo 2016; Cristiano et al. 2017; Foresti and Sanfilippo 2017; Marcuzzo and Rosselli 2018), adding to the hitherto sparse literature on the subject (Davenport 1975; Chua and Woodward 1983a; 1983b; Pierce 1993; Mini 1995), besides of course the editorial notes in Vol XII of Keynes'sCollected Writings (Moggridge 1983), which still remain the main and most authoritative reference.

A newer and more reliable description of Keynes's dealings has thus begun to emerge, assessing his performance as an investor as superior but not as stellar as had previously been believed.

Granted that Keynes fared quite well in shares, overall evaluation of Keynes's per- formance is still lacking.

formance is still lacking. We have partial results, but no complete and detailed analysis has been made of his investments as a whole.

Closer examination of Keynes's dealings in stocks for King's College has shown that in the earlier period of his bursarship Keynes came short of the market performance, and that he did not significantly outperform the market until he changed his strategy in the early 1930s (Chambers and Dimson 2013). Over the whole period, the annual performance of the funds over which Keynes had complete control amounted to +16.0 per cent, against +10.5 per cent of the market index (Chambers and Dimson 2013; 2015).

As far as Keynes's dealings in currencies are concerned, Accominotti and Chambers (2016) conclude that Keynes's discretional and fundamentals-based strategy on the whole failed to match the returns to rules-based strategies. Their test included carry trade (which borrows in low-interest-rate currencies to invest in high-interest-rate currencies), momentum (which consists in being long on past winners and short on past losers), and value (in which the investor is long on currencies that are undervalued in terms of purchasing power parity and short on currencies).

terms of purchasing power parity and short on overvalued currencies). Keynes only

managed to beat the carry trade, but not the momentum strategy, during the 1930s.

On commodity futures and options, Keynes achieved mixed results. During the 1920s, he made some profits in American Cotton (Cristiano and Naldi 2014) and Tin futures (Marcuzzo and Rosselli 2018), but he also incurred severe losses in rubber (CWK XII, p. 15), while options gained Keynes a total return on investment of 3 per cent over 12 years of activity. Taking the average yield of consols as a benchmark, Marcuzzo and Sanfilippo (2016) have shown that this was no great performance: Consols averagedd 4.5 per cent during the 1920s and 3.5 per cent in the 1930s.

Moreover, two other aspects of his activity as speculator have undergone scrutiny, namely his investment philosophy and its relationship with his economic theory. While the former has been thoroughly analysed, especially by Wasik (2013) and Woods (2013), we believe there is still more to be done on the latter.

Vol. XII of the Collected Writings contains a long series of public speeches and private correspondence in which Keynes promoted ex ante, or justified ex post, his investment strategy. During the 1930s in particular, the latter case became predominant. As the stock-market portfolios of the National Mutual and the Provincial fre- quently began to incur serious losses.

quently began to incur serious losses, Keynes had to justify his choices. This inevitably led him to produce a considerable flow of letters, memoranda and post mortems on investments, which Moggridge edited in 1983. Together with the more colloquial and relaxed post mortems on King's accounts, also included in vol. XII, these papers remain the main source for most of the contributions on Keynes's investment philosophy (Pierce 1993; Mini 1995; Holder and Kent 2011; Woods 2013).

In this paper we wish to contribute to this growing literature by filling some of the gaps, especially in relation to Keynes's investment philosophy and economic theory. To do so we undertake a more comprehensive review of the available evidence, drawing on some unpublished sources which have not yet been fully exploited.

2 KEYNES'S PRONOUNCEMENTS ON INSTITUTIONAL AND PERSONAL INVESTMENT STRATEGIES

It is a reasonable assumption that Keynes's approach to his private investments often

reflected the choices he made also as an institutional investor. There are, however, limits to the extent to which we can deduce Keynes's personal investment strategy from his institutional investments. One reason is the different time horizon of Keynes the individual investor vis-à-vis the institutional manager. Another is that, King's probably being an exception, Keynes could not be as autonomous in the management of others' funds as he was in his personal portfolio. Finally, there are two classes of investments that Keynes practised, namely commodities and currencies, which were outside the scope of at least some of his institutional dealings, although, as the correspondence with Kahn shows, not in the case of King's as far as commodities were concerned. Currencies and commodities remained outside the portfolios of the National Mutual and the Provincial, however. Nevertheless, the papers and correspondence of Keynes as an institutional investor remain a source of first-hand information.

An early experiment emerging from these papers is to be seen in the 'credit cycle' strategy.

strategy elaborated in collaboration with Falk and explained in the prospectus of the Independent Investment Company of January 1924 (CWK XII, p. 33). This strategy

started from the presupposition that fluctuations in the relative values of ...securities general-ly and of ordinary shares are all affected by a periodic credit cycle. Changes in the short-period rate of interest affect the value of long-dated securities to a greater degree than should strictly be the case, with the result that considerable profits can be made by changing from one class to another at the appropriate phases of the credit cycle. Similar periodic changes also take place in the relative values of money on the one hand and of goods and real property on the other, which are reflected in the relative values of bonds and shares ...so that here also the same principle of changing from one class to another at appropriate times can be applied. (CWK XII, p. 33)

1

In the same year of 1924, Keynes was popularising a new approach to institutional investors called 'active investment policy'.

investors called 'active investment policy'. In an article on 'Investment policy for insurance companies'(CWK XII, pp. 240-244), published in May in The Nation and Athenaeum, he argued that '[t]he wise investor must now doubt all things, and constantly revise his ideas in accordance with changing events in the political world'. Subsequently, the annual speeches as President of the National Mutual became the main vehicle for these ideas. The 1928 speech contains a clear definition of active investment policy, borrowed from a report of the Carnegie Corporation.

The funds of a great endowment can be kept intact only by a systematic revision

month by month of all securities of the endowment and by a continuous process of sale and exchange as circumstances may affect the financial soundness of this or that security' (CWK XII, p. 155).

While this 'active' policy was put into practice, Keynes grew firmly convinced that it was necessary to reform the portfolios of the companies in which he was involved. The quota of fixed interest securities had to be reduced, and that of equities enlarged. Speaking as President of the National Mutual in January 1928, encouraged by the good results he was achieving, Keynes proudly presented the new approach: 'We have been pioneers -he said -in the practice of employing a substantial part of our funds in ordinary shares'(CWK XII, p. 155). He explained that 'the centre of gravity of business, and therefore of investment, is not where it was [before the war]', and that

1. Much later, Keynes spoke somewhat contemptuously of this strategy. In a letter to Kahn of 5 May 1938 he wrote that he had 'seen it tried by five different parties ...over a period of nearly twenty years'without 'a single case of success'(CWK XII, p. 100).

However, which these 'five parties' may be is hard to tell, nor is it clear for how long and how much this strategy was actually followed by Keynes himself. Moreover, as we will see, the cycle investment left some traces also in Keynes's dealings during the 1930s. Nicholas Davenport, who became a member of the National Mutual board in 1932, later recalled: 'In the money and bond markets Keynes was able to apply his professional knowledge as an economist and monetary expert. The National Mutual would place its money on "the street "on a day-to-day basis when some crisis had driven the money rates sharply upward. Then it would move into the government bonds market when it foresaw money rates turning downwards. Finally, it would gather in its capital profits when it considered the gilt edged market had reached its peak .... Keynes ...was something to persuade the actuaries of the life offices to keep equities in their portfolios as a fixed and permanent proportion of their assets and to contem-plate 'switching'not only when management problems arose but when economic trends pointed to a "bear "market'(Davenport 1975, pp. 226-227).

Sticking to the orthodox and restricted range of fixed interest securities would mean 'living in a backwater'.

'living in a backwater'. Besides the traditional fields of railways and public utilities, usually within the empire, new opportunities were emerging. He mentioned the oil business, the tea, coffee and rubber industries, and 'the ordinary shares of companies overseas, particularly in the United States'(CWK XII, p. 157). In round figures, there

were '250 companies with a total ordinary share market capitalization of about £1,500,000,000', which represented, in Keynes's view, 'the live large-scale business and investment world of today'(ibid.).

There is no evidence that the crash of 1929 changed Keynes's opinion that investing in equities was the right policy.

in equities was the right policy, but it certainly made it much more difficult to make it palatable to the Boards of the National Mutual and the Provincial. From 1930 onwards, the National Mutual speeches showed the need to defend, rather than the will to promote, equity investment. The same happened with the Provincial, as the sample of Keynes's correspondence with F.C. Scott reproduced in CWK XII clearly shows.

The only exception is the management of the funds of King's College.

Throughout a period of about 25 years as Bursar, Keynes enjoyed a privileged position that was largely denied to him as member, or even chairman, of other Boards. In practice, he was given carte blanche in the administration of a considerable amount of money over an indefinite period. This gave him the opportunity to pursue the strategy of investment that best reflected his opinions. While the traditional investment of King's, as well as other colleges, was mainly in real estate and gilt-edged securities, Keynes created and managed for his college a strong position in equities, a policy he tried to maintain also after 1929. Incidentally, the same strategy is still of interest today as a pioneer example of what became customary in college finance only in the second half of the last century (Chambers and Dimson 2015). But in the context of the post-1929 slump, the most salient aspect of Keynes's stock-market strategy was his resolution to avoid large liquidation of shares during a period of a prolonged and dra- matic fall in market prices.

matic fall in market prices. This is evident in the administration of the King's College funds -Chambers and Dimson place great emphasis on this point -as well as in National Mutual speeches and the published correspondence with other Board mem- bers during the 1930s.

bers during the 1930s.

However, this trading behaviour was probably the result of Keynes's rethinking at an earlier stage.

earlier stage. The earliest evidence of Keynes's preference for equities is the review of Smith (1925), published in May 1925 (now in CWK XII, pp. 247-252). Here Keynes argued that investing in equities was investing in real values instead of money values,

and moreover in a world in which money depreciation was supposed to be the most predictable outcome. Another reason Keynes insisted on was that well-managed firms do not usually distribute all their profits to shareholders. Rather, they prefer to reinvest this money into business. Thus there is an element of compound interest operating in favour of a sound industrial investment'(CWK XII, p. 250, emphasis in the original). Chambers et al. (2015a; 2015b) have shown how, as time went by, the selection of the most 'sound industrial investments', the ones with the best long-term outlook in spite of their low current market evaluation, became Keynes's particular hobby in King's administration. This entailed that he put relatively large sums of money into relatively few assets, thus betting on his ability to pick out the most out the most undervalued assets while eschewing diversification. Accordingly, the investment policy of Keynes as a mature investor has been described as rather idiosyncratic, and therefore scarcely compatible with collective management.
Turning, now, to his personal investment, at an earlier stage Keynes's personal investments had, as we have seen, been largely in commodities and currencies.

In these markets, the kind of 'cycle investment strategy'that left traces in Keynes's papers around 1924 was a natural approach. It has been observed (Accominotti and Chambers 2016, pp. 360-361) that Keynes's exchange speculation was based on a 'discretionary' analysis of 'macro-economic fundamentals as expected changes in official interest rates, the level of European reparations, international trade and capital flows, and the inflation outlook when making his currency forecasts'. Moreover, something similar has emerged upon closer examination of Keynes's dealings in commodities. There is evidence that Keynes collected detailed information about all the commodities he traded, and that, at least throughout the 1920s, he tried to predict the price trend of each commodity in the context of his broad outlook on the trade cycle. It is even possible that their experience in these markets had some influence on Keynes and Falk when they drafted the Independent prospectus in January 1924, and that they were just trying to extend their experience in commodities and foreign exchange to the bond and share markets. In any case, that Keynes could invest on the basis of his predictions about the cycle looks like a natural spin-off

of his economic theorising. As he wrote in the Tract on Monetary Reform, 'the price level is not mysterious, but is governed by a few, definite, analysable influences'(CWK IV, p. 68), which implies that it is at least to some extent predictable.

The Tract was published in 1924. The ensuing years were spent by Keynes in the long preparation of A Treatise on Money (1930), which presents a far more detailed theory of the credit cycle. Even more than the Tract, the Treatise suggests that profitable investments could be based on advanced theoretical knowledge. On the one hand, the basic ideas are simple, which is consistent with the view of cycles as predictable phenomena. Keynes identified the level of investment as the main drive of fluctuations and the level of the rate of interest as the main determinant of investment. Following Wicksell, a market rate of interest below the natural rate of interest will cause a rise of investments and therefore an upward trend of the economy. The opposite happens when the market rate is above the natural rate. On the other hand, as noticed by Moggridge (1992, p. 486), 'Keynes added useful complications at every turn', thus making the application of the theory difficult enough to restrict the number of investors who could actually exploit it to their advantage. Keynes's negative comments on cycle trading as put forth in the above-mentioned letter to Kahn of 1938 (see footnote 1) suggest that difficulty of application eventually prevailed over the relative simplicity of the logic underpinning the theory.

Furthermore, Accominotti and Chambers (2014) note that Keynes also 'attempted to exploit information gleaned during his meetings with diplomats, bankers, and stakeholders involved in important currency discussions', which has more to do with the exploitation of specific information advantages than with some superior ability in macroeconomic analysis and prediction.

Turning to commodities, we find a similar picture. In his tin dealings, Keynes attempted to exploit his personal connections in the City (Cavalli and Cristiano 2012; Marcuzzo and Rosselli 2018). Moreover, the significant investment in tin shares during the same period in which Keynes accumulated a considerable open interest in this commodity probably depended on the same information. Again, Keynes motivated his large investments in South African gold-mining shares with his personal exchanges with the manager of a top firm in that business (Henry Strakosch; see Keynes's letter to Scott of 15 August 1934 in CWK XII, pp. 55-57).

As Keynes grew older and his network of personal contacts widened, this kind of strategy became more practicable. In a world devoid of any serious rule on insider trading, Keynes had no need to hide the fact that he had influential friends in the City. In fact, as so much of the correspondence in the Keynes Papers reveals, in most of the cases this kind of information had nothing to do with inside trading. Rather, Keynes widened his relationship with business houses and professional investors, with whom he could profitably exchange information as well as opinions on a wider range of specific investments.

Along with the 'humbling déjàvu of having nearly lost two fortunes' upon which Wasik (2013, p. 84) places much more emphasis, this could have contributed to making the abandonment of cycle trading quite a natural development, in Keynes's institutional investments as well as in his personal dealings. However, the unpublished material, mainly correspondence, which Keynes held with City people, brokers and friends, has not been fully explored.

3 KEYNES'S MAIN ADVISORS

According to a recent study, the network of his 'personal contacts from Keynes's time at Eton College, Cambridge University, the Treasury during World War I, and from public life consisted of 7,632 people' (Eldridge 2012, quoted in Chambers and Dimson 2013, p. 225).

Among these myriad contacts, four names stand out -Oswald T. Falk, Rupert Trouton, Walter S. Case and Richard F. Kahn -while there is a lesser-known one that probably deserves to be added, namely Francis C. Scott.

Oswald Toynbee 'Foxy'Falk is the man with whom Keynes started up in business on a larger scale. Friends since their days at the Treasury, they cooperated in the creation of a network of financial endeavours in time of peace. After the First World War, Falk became a partner in the stockbroker firm Buckmaster &Moore, through which Keynes would manage part of his dealings, and preceded Keynes on the Board of the National Mutual. Then, as we have already seen, the two men became cofounders of the Independent, the P.R. Finance Company, the A.D. Investment Trust and the Syndicate. In the mid 1920s they shared the same enthusiasm for 'cycle investment', the idea being that it was possible to make money by predicting economic trends, buy-

ing assets when prices were rising and selling them at the beginning of the downturn. The end of their collaboration, if not of their friendship, has recently been attributed to the failure of this strategy and the consequent crisis in which most of their joint endeavours fell with the 1929 crash (Wasik 2013, p. 70). Another interpretation also gives prominence to their opposed views as to the industrial future of England, and therefore on the advantageousness of investing in the UK market rather than in Wall Street (Millow 2012, p. 403). A key episode occurred when Falk wrote a letter to The Times newspaper in 1930, urging British investors to fly to the US market as there was no future in London, and Keynes replied to him in turn with a letter to The Times. Keynes certainly did so for patriotic reasons, and possibly in consideration of his public standing. Whether he actually believed that Falk was wrong is another matter. On the one hand, in the ensuing years he began to invest in Wall Street on a a larger scale (Chambers and Kabiri 2016; Cristiano et al. 2017). On the other, it is also true that he did not abandon the British stock market.

Rupert Trouton worked with Keynes for the Government during the First World War, was his student at Cambridge, and had a lot of dealings with him when he was at Buckmaster &Moore and Laurence, Keen &Gardner, which were King's main broker firms. It was Trouton who, in 1921, introduced Keynes to metal options (Marcuzzo and Sanfilippo 2016). He was a cofounder with Keynes and Falk of the

AD Investment Trust and the P.R. Finance Company, where collaboration between Keynes and Trouton was very close. Trouton was able to reverse the fortunes of the company after the bad years 1928-1932, and liquidation of the company in 1934 brought profit to the shareholders (Basberg 2015). When Trouton set up his own company, Hector Whaling, in 1928, Keynes remained invested in it, both for himself and for the College throughout its ups and downs to the very end. Trouton, like Kahn, was an economist trained by Keynes at King's and their discussions over investment policy must surely have had that particular slant which was possibly lacking in other relationships.

Walter Summerhayes Case was an 'American investment banker. Founder, 1916, president and director of Case, Pomeroy &Co., Inc., a private New York investment company with a specialised research organisation' (Skidelsky 1992, p. 690). Since the

early 1930s, Case (and his business house) had become Keynes's privileged source of professional analysis and information on specific investments and classes of investments (at least for the US market). This lasted until October 1937, when Case com- mitted suicide.

mitted suicide. As Keynes put it to Kahn, '[i]t was nothing to do with finance (he had been mainly bearish, particularly in commodities, and was largely out of markets); and I think it was probably due to a recurrence of health trouble which he confided to Lewis]

2

(and few others knew I think) ..., but he thought that he had completely recovered'(Keynes to Kahn, 7 October 1937, in Richard Kahn Papers, King's College, RFK/13/57/252-253).

On 22 December 1937, The Times published an obituary of Case by Keynes, now reproduced in the Essays in Biography (CWK X, pp. 326-327). Keynes emphasised Case's view of `the purely financial and Stock Exchange side of his business'as a means to the end of 'the active development of the world's resources', his 'fanatical enthusiasm for the application of science to business affairs', and the 'lavish[ness] in his expenditure on obtaining the best possible assistance and advice'(ibid., p. 326).

Unfortunately, Keynes also had to remark that Case 'never wrote a letter or put a pen to paper' and that 'he was not a scientist.

to paper'and that 'he was addicted to the long-distance telephone even beyond ordinary American usage'(ibid., p. 326). Despite Keynes's dislike for telephone conversation,3 this did not prevent Case from becoming 'The American Financer with whom I was most intimate and on whose advice I most relied'(KP BM/3/157), 4

as Keynes wrote to Francis C. Scott on 25 April 1939. A few weeks before Case's death, Keynes reported to Kahn that '[i]n light of his [Case's] opinion', he had just ordered 'for my-self and the College' 'some more'of 'Homestakes'.5

The same letter includes some examples of the kind of information that Keynes received from his friend and the use he made of it:

U. S. Smelting.

7

He remains of the same opinion, though he is annoyed that apparently there is not as yet any material increase in the output of lead. The stickiness of the price is due, he

2. Probably Sir Alfred Edward Lewis, director of the National Provincial Bank and member of the Economic Advisory Council (Moggridge 1992, p. 888).

3. See J.M. Keynes, 'The nuisance of a telephone. To the editor of the New Statesman, 23 December 1922', in CWK XVIII, pp. 100-101.

4. Permission to publish from Keynes Papers, King's College Library, Cambridge, UK is gratefully acknowledged.

5. Homestake Mining Company, owner of a gold mine in Dakota, was listed on the New York Stock Exchange.

6. On Keynes's investment in Wall Street, see Cristiano et al. (2017).

7. United States Smelting Refining &Mining Co says, to sell by a large estate, which has to realise the money, and puts stock on the market whenever the price crosses 90.

...United Gas and Electric Power and Light. He had no information about the arrangements for financing, but he is confident that they are not yet at an end of their important oil-field discoveries. They have a major field in the Rodessa and a minor field in the Sligo, but they will be extremely unlucky if they do not find at least one more major field, and they might do better still. I enclose a cutting from the Financial News, in case you have not seen it. Generally speaking he was just as keen on utilities as we are. (Keynes to Kahn, August 1937, in RFK/13/57/213-216)

Richard Kahn was, as we know, Keynes's 'favourite pupil', a companion travelling with him on the road towards The General Theory, and a friend and collaborator of Keynes's in several academic, personal and financial matters; in his capacity as Second Bursar of King's and Director of the Tilton Company, he assisted Keynes in several investment decisions and shared with him assessment of market conditions, upon and they acted both for their own portfolio and for the College. So much is testified by the correspondence between the two, consisting of 611 letters, only 68 of which were published in the CWK (Marcuzzo 2005). Almost half of the surviving letters are from 1937 and 1938, as from the time when Keynes had fallen seriously ill in the Spring of 1937 and for months all financial and academic matters were handed over to Kahn (see Fantacci et al. 2010).

What these letters tell us is that on each issue they exchanged detailed information, comparing their respective evaluations and assessments, on commodities, American

and British shares, bonds and currencies. Keynes taught Kahn -who was in any case by nature so inclined -to keep updated with detailed knowledge of every aspect of the matter in hand. On the other hand, he was also giving Kahn tips based on his wisdom as investor, as the following excerpts illustrate:

[A]s you are discovering, [dealing in commodities] is a business which needs hard work; and it does not turn out right over a period of years unless one attends to the details which, cumulatively, add up to quite a lot. But it is a pure game and should not use time available for serious tasks. (Keynes to Kahn, 14 July 1937, RFK/13/57/193-194)

For several years I have always felt during a recession that it was worth hanging on, and, provided one's cover position was all right, all one had to do was to wait; so that if I felt the cover position was quite safe, I didn't bother. But today I don't feel like that. I don't want to have a big loan, even though the cover position is perfectly good. I've not got to the point of being a bear, but I am much more disinclined to be a bull on borrowed money. (Keynes to Kahn, 2 September 1937, RFK/13/57/231-232, reproduced in CWK XII, pp. 24-25)

The correspondence with Francis Clayton Scott is also very revealing in similar respects. Scott was born in 1881 and educated at Bedales and Oriel College, Oxford (Moggridge 1992, p. 902). In his capacity as President of the Provincial Insurance Company, he was in constant touch with Keynes. The sample of this correspondence that is reproduced in vol. XII of the CWK suggests that the two men disagreed on investment policy, Scott being more sensitive than Keynes.

investment policy, Scott being more sensitive than Keynes to short-period fluctuations of asset prices.

However, the relationship with Keynes was much more friendly, and their correspondence more constructive, than the published material would suggest. Part of this correspondence will be considered in the next section, together with other evidence -mainly unpublished, drawn from the Keynes Papers -that can add something to our understanding of the developments in Keynes's thinking

4 DEVELOPMENTS AND CHANGES IN KEYNES'S INVESTMENT PHILOSOPHY RECONSIDERED

There is general agreement in the recent literature that the turning point in Keynes's

stock-market investments came with one major change that occurred in the early 1930s. It is a well-established fact that, by this time, Keynes had decidedly shifted to the kind of buy-and-hold approach that is well exemplified in his college administration, and that Keynes based this strategy on limited diversification and a highly idiosyncratic selection of a restricted set of shares. Also, Marcuzzo and Sanfilippo (2016) found that Keynes basically gave up his dealings in options in the early 1930s, which can also be interpreted as a break in his style of investment.

Less clear is why, and exactly when, Keynes changed his mind, and how the new stock-market strategy relates to the parallel investment in commodity and exchange markets. Of course, this is not something that can be divined from Keynes's utterances, however numerous they may have been. Without all-inclusive study of Keynes's portfolio and its evolution, no final conclusion can be reached on this matter. One point we might venture at the present stage is that the distinction between the two strategies was probably more blurred than is sometimes suggested.

For instance, a letter from Falk to T.J. Carlyle Gifford (co-director at the Independent Investment Company) sheds some light on how the cycle investment strategy may have been carried out in practice. The letter was sent on 8 February 1924. (8)

8. A copy of this letter was forwarded to Keynes, among whose papers it remained (see KP IIC/1/1-5).

Falk explained his dislike of an operation that the two were considering. What this investment was is not clear, but Falk did not like it for two reasons. First, he found it 'too great a departure from the credit cycle plan'. Second, the proposed purchase was too small. Falk's argument was that their policy at the time was to invest a certain amount of money in a limited number of assets. Otherwise, they would have had to 'acquire information'on too many assets. This seems to suggest that, even at the beginning of the cycle trading period, the strategy was not altogether top-down but at least in part bottom-up.

Evidence and common sense concur in suggesting that Keynes never dropped his own view on general business trends just because he had abandoned cycle trading. By the time he had switched to the bottom-up approach, Keynes's opinions as a profes- sional economist were purchased (at least in part).

sional economist were purchased (at no trifling cost) by business houses which employed them as a basis for their investments. There is no reason to suppose that he failed to employ the same analyses in determining his own investments and those of the Provincial, King's and so on. Moreover, Keynes's correspondence with Scott shows that predictions about the cycle continued to find a place in Keynes's reasoning:

As regards buying some more railway shares, I personally entirely agree with you. Indeed, I am rather strongly in favour of them. They seem to me to be an almost ideal credit cycle security, in the sense of being good things to buy when one hopes it is somewhere near the bottom of the slump. (Keynes to Scott, 19 August 1932, KP PC/1/1/130)

What Keynes added along the way was a huge amount of detailed information on a selection of shares from a number of sources.

selection of shares from a number of sources, along with an ever more refined taste (developed by cross-examination as well as trial and error) for these sources of professional (as distinguished from confidential) information and business analysis.

On 10 October 1935, for instance, Keynes wrote a letter to Scott containing four pages of detailed analysis of Austin Motors.

pages of detailed analysis of Austin Motors from both the industrial and financial points of view (KP PC/1/4/123-127). This is only one example of Keynes's detailed study of one of his 'pets', but an entire paper could be dedicated to his long disquisitions with Scott on Austin Motors shares and a few other equities.

As mentioned above, Scott was not as inclined as Keynes to eschew the speculative mentality. Nevertheless, their dialogue always remained on a constructive basis, apparently because Scott adhered to the same idea of 'active investment policy'supported by Keynes. This emerges in all the evidence on perusing not only the letters but also the accompanying material and the related correspondence.

accompanying material and the related correspondence, as now collected in the Keynes Papers. The bulk of this material amounts to hundreds of documents of a sundry nature, but all relate to the same need, which was to keep up to date with

the markets by constantly gathering evidence.

In this respect, the correspondence with Scott sheds light on a revealing though lesser-known aspect of Keynes's work.

known aspect of Keynes's business relationship with Walter Case. When D.S. Roswell, a former employee of Case, Pomeroy and Co., was about to set up a new business firm after Case's death, Keynes wrote to Scott about the proposed collaboration, describing Roswell as Case's 'principal expert in reporting on the intrinsic value and prospects of American companies' (letter dated 25 April 1939, now in KP BM/3/157-158). He told Scott he had known Roswell 'for a long time past', and that 'he had more faith in his opinion ...than anyone else I know'(ibid.), but also that, unless Keynes could meet Roswell during one of his American visits, this opinion was usually filtered through Case. Now Roswell was 'prepared to offer his services to a limited clientele'(ibid.) for an annual fee of £2500 which, Keynes proposed, could be divided between the Provincial, King's College and himself. (9)

A passage in Keynes's letter to Scott is particularly revealing of the kind of information that Keynes was looking for as a mature investor at the apex of his career:

What he does is to make incredibly careful studies of the intrinsic value and long period prospeccts of the main American securities, the sort of study which it is unfortunately impossible to have made in this country for lack of data, but which can be made in America, especially in the case of the leading utilities. (Keynes to Scott, 25 April 1939, KP BM/3/157-158)

No doubt this is further proof that Keynes's investment philosophy had definitely turned against cycle investment and in favour of stock-picking value investment.

In the context of the correspondence with Scott, however, another interesting possibility this passage suggests is that Keynes was looking for a kind of business information and analysis that could be complementary to his own information and analysis as an economist. Connected to this is the fact that the choice between different markets sometimes depended on the relative costs and availability of information, Wall Street being at an advantage over London in this respect.

Taking a broader view, what emerges from Keynes's relationships with his major advisors is that his individualism as an investor may have been somewhat exaggerated.

Keynes is frequently quoted for his observation that '[i]t is astonishing what foolish things one can temporarily believe if one thinks too long alone, especially in economics' (Preface to The General Theory, CWK VII, p. xxiii). Apparently, for Keynes, the same applied to business too.

9. In the end a deal was reached, as Keynes announced to Roswell on 10 May 1939 (CP BM/3/171)

True enough, Keynes sometimes complained of the advice received from his correspondents. (10)

On the whole, however, Keynes relied on them. In the same vein, the fact that he sometimes expressed dislike for Board management may have concealed another fact, namely that he did like to have frequent exchanges of information and qualified opinions with a select range of friends and collaborators. What the Keynes Papers abundantly show is that, for Keynes, investment was a time-consuming activity in which building up a network of reliable connections and collecting sound, relevant information was a costly but decisive task. This he made quite clear in a letter to Scott:

It sometimes seems to me that apart from the noble army of investors who never read the newspapers I am almost the only person left who has an investment rather than a speculative mentality! On every Board I sit on the great majority are influenced far more by the daily fluctuations which they read of in the newspapers than by reasoned calculation of yield or ultimate prospects. (Keynes to Scott, 7 June 1937, KP PC/1/4/306)

Given this approach, Keynes reaped considerable economies of scale through his participation on several Boards, which might explain why he stayed on them even though he was wont to say that he could not stand them:

The danger of Board management, against which one has to be on one's guard, is lest one should succeed in persuading the Board rather against its better judgment in the first instance, and then have to suffer the penalty of their faint-heartedness at a later date, just when the virtues of continuity of mind are most required if one is to be successful in the long run. (Keynes to F.C. Scott, 29 November 1933, in CWK XII, p. 65)

Connected to this is the extent and variety of information that could be relevant for a man who was a professional economist but not a full time businessman (like, for instance, Case). This is especially true of the 1930s. As an investor, Keynes had

now grown up from the amateurish style of the early 1920s, when Trouton and Falk had to brief him on cotton futures or metal options. But he was also engaged in a considerable number of parallel activities as an economist.

As it turns out, Keynes's portfolio choices may have been the result of the juxtaposition of his general vision and systematic analysis of the entire economic system with the information he could actually obtain -this latter element being, at least to some extent, more a matter of circumstances than of deliberate choice. The evolution of Keynes's economic thinking ran parallel to the development of his business skills, the increase in his business contacts, and a process of selection of these sources of information.

5 THE INVESTOR AS ECONOMIST AND THE ECONOMIST AS INVESTOR

It is more than likely that there was some connection between Keynes's changing investment policy and the developments in his economic theory. His views on

10. As, for example, in the following excerpt about the losses incurred by the National Mutual: 'You will notice that these are practically all specialties and rather obscure concerns, mostly bought on private advice. Omes was due to Trouton: Carbo Plaster and South African Torbanite to Falk; Enfield Rolling Mills and Grand Union Canal to [W. Harold] Brett. I am sure experience shows that private and personal recommendations of this class of security tend to turn out wrong in the long run' (JMK to F.C. Scott, 7 June 1938, in CWK XII, p. 66). W.H. Brett was one of the brokers employed by the Provincial

speculation changed over the years as his theory developed and his practice as speculator improved.

The first instance was in 1910, when he was a lecturer at Cambridge and had practically no experience in the Stock Exchange. In his lectures Keynes distinguished guished between speculators, who base their decisions on the possession of 'superior knowledge', and gamblers, who just take more or less calculable risks, as in the game of roulette. Superior knowledge confers the speculator with an advantage over the market. To Keynes this is a matter relevant not to measuring comparative success in gambling and in speculation, which may be dependent on other factors, but to evaluating the nature of the action in the two cases. Unlike speculation, gambling is not reasonable because it is a behaviour which has no basis in knowledge, notwith-

standing the fact that a gambler may at times be a winner and a speculator a loser (see KP/UA/6/3 and Carabelli 1998).

The next phase in Keynes's thinking -as he became more closely acquainted with the working of markets -was the analysis of speculation in futures (currencies and commodities) presented in his 'The forward market in foreign exchanges'(1922), incorporated in the Tract of Monetary Reform (1924; CWK IV) and in his 1923 article 'Some aspects of commodity markets'(CWK XII, pp. 255-265). The points made there were reiterated in A Treatise on Money, where he gave a more refined version of his theory.

Future contracts are described as a form of insurance policy against price fluctuations.

tions. By stipulating these contracts, producers (consumers) of a commodity for which a future market exists fix in advance the price of a future sale (purchase), thereby freeing themselves from the risk of a price decrease (increase). Keynes assumes that it is mostly professional speculators, generally less risk-averse than producers, who make forward purchases. The prospect of gaining from price changes by buying forward in anticipation of a price increase would eventually allow speculators to resell at a profit on maturity of the forward contract (Fantacci et al. 2010).

Speculators who enter into forward contracts do not have firm expectations of price changes, and hence of windfall profits.

changes, and hence of windfall profits, but, by providing an insurance against unex- pected price changes, they enter the

pected price changes, they enter the market for the gain they stand to make, rewarding them for the risk price changes entail. Thus, in Keynes's new view, the speculator's ability to forecast the future through superior knowledge is downplayed. He is not a prophet'(CWK XII, p. 260), that is, someone who can anticipate price movements more accurately than other actors, but rather a 'risk bearer'(ibid.).

This new view did not rule out the importance for a speculator of being acquainted with the working of the future markets, nor of being knowledgeable about specific commodities or currencies, on which the dividing line between a gambler and a professional trader is drawn. In fact, in the transition between the Treatise and The General Theory, the original idea of rational speculation based on knowl-

edge was incorporated into other terms, like 'investment' or 'enterprise'.

As Keynes became more and more an investor in shares, rather than a speculator in commodities -sometime during the period 1933-1934, which also saw his 'revolution'in economic theory in progress -his views on speculation extended to the idea of conformative behaviour based on some tacitly established convention.

For a speculator is a man who anticipates the behaviours of other speculators, so that if all speculators have the same anticipations, all of them will, temporarily, be right; and only when the music stops -for musical chairs is the game which speculators play with one another -will someone find himself without a seat. ('The kaffir boom', February 1933, in CWK XXI, pp. 227–228)

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This is the view of speculation that found definitive exposition in chapter 12 of The General Theory, which hinged upon the opposition between speculation as the attempt to adapt to other people's opinions -no matter whether right or wrong -and enterprise as a behaviour based on actual knowledge of fundamentals.

This entailed transition from a view of speculation as a form of rational and socially sound economic behaviour, most plausibly rooted in the cross-fertilization between the Marshallian explanation of the subject and Keynes's own ideas on rationality, to a view of speculation as possibly rational from the individual agent's viewpoint, but antisocial. As Dardi and Gallegati (1992, pp. 582-583) argue, the view of speculation as distinguished from enterprise, and the distinction between the professional speculator and the amateur, are common to Keynes and his master. Unlike Marshall, however, Keynes grew convinced that the information advantage of the speculator (relative to the amateur) was not about fundamentals. If any advantage there was, it was about the average opinion of the market.

Viewed from the standpoint of his investments, which was so very fundamentals-oriented, speculation may have become for Keynes not just an alternative to, but also an obstacle in the way of, sound investment, or enterprise.

6 CONCLUSIONS

In this paper we have reviewed the growing literature that has in recent years taken up a somewhat neglected aspect of Keynes's life as speculator and investor. In particular,

we have pieced together the evidence collected on his performance, pointing out that much more needs to be researched before we can conclude that it was in fact 'stellar', as the traditional account has it.

As far as his investment philosophy is concerned, there seems to be a general consensus in describing it as characterised by two distinct phases: the first, from the early 1920s to the early 1930s, guided by the 'credit cycle' approach; the second, which Keynes then turned to and pursued to the end, a bottom-up strategy. Cycle trading assumed that assets in general are systematically under- or overvalued at different stages of the trade cycle, and that decisions to sell or purchase should therefore be more on general conditions than on specific knowledge of individual assets and their fundamentals. By contrast, the second approach assumes that with closer examination of specific assets and their fundamentals it is possible to pick out the best of them in terms of prospective yield and/or current price.

While we agree that there seems to be a clear break in his investment behaviour, we would hesitate to attribute it to complete abandonment of the credit cycle approach; indeed, we believe he never completely relinquished it, even during the years when he focused on picking the shares which promised well in terms of future yields.

On the basis of some unpublished material, in particular the correspondence with Scott, we have, we hope, helped to fill in the picture of how Keynes formed his opinions, what information he was seeking and on whose advice he mostly relied. In addressing the evolution of his trading behaviour, we have tried to match it with the developments in his economic thinking in general and on speculation in particular.

What emerges from all this is that Keynes never ceased to be first and foremost an economist who kept sight of the complexity of factors behind the surface of price changes; while he progressively

changes; while he progressively lost confidence in the ability to predict their course in the short run, he remained confident that study of the fundamentals of the economy and of what underlies the individual assets would provide a reasonable basis for a rational, and in the long run at least, successful choice.

11. Kaffirs' was the name given to the South African gold shares quoted on the London market.

When South Africa abandoned the gold standard in December 1932, the South African pound depreciated, thus boosting the local currency receipts of the South African gold-mining companies. This led to a boom in 'Kaffirs' in London.